Mohammad Ali Shah

Complexity Reduced Soft-In Soft-Out Sphere Detection for Multi Antenna Systems

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Complexity Reduced Soft-In Soft-Out Sphere Detection for Multi Antenna Systems

Mohammad Ali Shah

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zur Erlangung des akademischen Grades eines

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Abstract

The ever growing demand of high data rates in mobile communication systems calls for methods which allow to use the radio frequency spectrum as efficiently as possible. Future mobile communication systems most likely will involve multiple-input multiple-output (MIMO) techniques in combination with high order constellations, raising the amount of transmitted data per channel usage in order to enhance spectral efficiency. However, due to non-orthogonality of the transmission channel, this performance improvement comes at the cost of increased computational complexity in the receiver. One of the main challenge is the computationally intense task of MIMO detection use to separate the spatially multiplexed data streams.

Motivated by the tremendous gains in turbo channel decoder, the concept of iterative processing has been recently extended to include iterations between MIMO detector and channel decoder. Soft-In Soft-Out (SISO) detectors, when concatenated with a channel decoder, can significantly improve the quality of wireless transmission by performing joint, iterative data detection and channel decoding through the exchange of soft information. However, soft information from channel decoder increases the search space and hereby the computational complexity of the tree search in SISO detector. Furthermore, the complexity of the optimal MAP detector grows exponentially with system dimensions. These considerations motivate the design of complexity reduced suboptimal detectors for iterative detection and decoding.

This thesis focuses on complexity reduction of the SISO detector at minimum performance loss and to enable high throughput detection. Detection algorithms based on depth first tree search enables MAP detection performance at reduced but still high complexity. This thesis introduces a noval method for complexity reduction of depth first search detector. Based on the analysis of the reliability information form channel decoder, predefinition of the bits values is enabled. This allows to reduce the search space and thereby ease the complexity of detector. In addition to this, parallel processing has been exploited in the target detection algorithm to speed up the detection and increase throughput. In order to evaluate the performance and complexity of hardware implementation, this thesis includes VLSI implementation of the processor model for soft-out MIMO detection. Synthesis results show that it is possible to achieve a high throughput compared to state of the art implementations with relatively small chip area. Final evaluation of the SISO detector is performed in a case study on MIMO detection for 3GPP LTE system.

Kurzfassung

Der stetig wachsende Bedarf nach hohen Datenraten in mobilen Nachrichtensystemen erfordert eine zunehmend effiziente Nutzung des Übertragungsspektrums. Zur Verbesserung der spektralen Effizienz werden zukünftige mobile Nachrichtensysteme voraussichtlich Mehrantennentechniken (Multiple Input Multiple Output - MIMO Techniken) einbeziehen um in Kombination mit hohen Konstellationsgrößen möglichst große Datenmengen pro Kanalzugriff zu übertragen. Aufgrund der Nicht-Orthogonalität der Übertragungskanäle sind zum Erreichen dieser Leistungssteigerung empfangsseitig jedoch rechenintensive Operationen notwendig. Eine der maßgeblichen Herausforderungen hierbei ist die aufwändige MIMO Detektion zur Separierung der räumlich überlagerten Datenströme.

Motiviert durch die enormen Gewinne von Turbo-Kanaldekodern wurden in letzter Zeit die Konzepte iterativer (Turbo) Verarbeitung so erweitert, dass diese auch Iterationen zwischen MIMO Detektoren und Kanaldecodern umfassen. Sogenannte Soft-In Soft-Out (SISO) Detektoren können hierbei in Verbindung mit geeigneten Kanaldecodern die Übertragungsqualität signifikant verbessern. Ermöglicht wird dies durch eine gemeinsame, iterative Detektion und Dekodierung und den Austausch von Zuverlässigkeitsinformationen (Soft-Werten). Ein Nachteil dieser SISO Detektion ist jedoch, dass durch die Zuverlässigkeitsinformationen des Kanaldekoders der Suchraum nachgelagerter Detektionen und somit der Rechenaufwand der enthaltenen Baumsuche vergrößert wird. Des Weiteren steigt die Komplexität des optimalen MAP Detektor exponentiell mit den Systemdimensionen. Abhilfe bietet die Entwicklung komplexitätsreduzierter suboptimaler Detektoren für die iterative Detektion und Decodierung.

Fokus dieser Arbeit ist die Komplexitätsreduktion von SISO Detektoren bei minimalem Genauigkeitsverlust zum Ermöglichen hoher Datenraten. Detektionsalgorithmen auf Basis von Baumsuchen erreichen die Genauigkeit der MAP Detektion mit reduzierter jedoch immer noch hoher Komplexität. Zur weiteren Komplexitätsreduktion wird im Rahmen dieser Arbeit ein neuer Ansatz für Detektionsalgorithmen der Tiefensuche vorgestellt. Aufbauend auf einer Analyse der Zuverlässigkeitsinformation des Decoders ist es möglich einzelne Bitwerte vor der Detektion festzulegen. Dies führt zu einer deutlichen Reduktion des Suchraumes und somit zu einer Verringerung der Detektionskomplexität. Eine weitere Steigerung des Durchsatzes wurde durch eine Parallelisierung des gewählten Detektionsalgorithmus. Zur Bewertung der Leistungsfähigkeit und Komplexität einer Hardwareimplementierung wurde zudem eine VLSI Implementierung des zugrundeliegenden Soft-Output Prozessormodells erstellt. Die Ergebnisse der Synthese verdeutlichen, dass, verglichen mit herkömmlichen Umsetzungen, hohe Datenraten mit relativ kleiner Chip Fläche erreicht werden können. Eine abschließende Bewertung des SISO Detektors wurde anhand einer Fallstudie für 3GPP LTE Systeme durchgeführt.

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Contents

List of Figures

List of Tables

Abbreviations

3GPP	3rd Generation Partnership Project
APP	A Posteriori Probability
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BICM	Bit-Interleaved Coded Modulation
BCJR	Bahl-Cocke-Jelinek-Raviv
CDF	Cumulative Distribution Function
CP	Cyclic Prefix
dB	deciBel
FER	Frame Error Rate
FFT	Fast Fourier Transform
FU	Functional Unit
i.i.d.	independent and identically distributed
IEEE	Institute of Electrical and Electronics Engineers, Inc.
Iter	Detector↔Decoder Iteration
IFFT	Inverse Fast Fourier Transform
L-values	Log-likelihood ratios
LSD	List Sphere Detektor
LTE	Long Term Evolution
MAP	Maximum A Posterior
MaxLogAPP	Max-Log aprroximation of APP
ME	Metric Estimation
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MUX	MUltipleXer
OFDM	Orthogonal Frequency Division Multiplexing
OP	OPeration
PBPTS	Parallel Bit Pruned Tuple Search
PCCC	Parallel Concatenated Convolutional Code

QAM	Quadrature Amplitude Modulation
QRD	QR Decomposition
SMEM	Scalar MEMory
SD	Sphere Detection
SE	Schnorr Euchner
SIC	Successive Interference Cancellation
SIMD	Single Instruction Multiple Data
SISO	Soft-In Soft-Out
SNR	Signal to Noise Ratio
SSD	Search Sequence Determination
SQRD	Sorted QRD
SBPTS	Sequential Bit Pruned Tuple Search
STA	Synchrone Transfer Architecture
STS	Single Tree Search
TS	Tuple Search
TS-SSD-ME	TS with Search Sequence Determination and Metric Estimation
VMEM	Vector MEMory
VLIW	Very Long Instruction Word
ZF	Zero Forcing

List of Symbols

Operations and Functions

$\mathbb{C}^{n\times m}$	Set of complex numbers of dimension $n \times m$
$\mathbb{R}^{n\times m}$	Set of real numbers of dimension $n \times m$
$\mathbb{Z}$	Set of integers
$\forall$	For all
$\in$	Belongs to
$\subseteq$	Subset
$\sum$	Sum
$\prod$	Product
Π	Interleaver
Π^{-1}	Inverse Interleaver
$\lvert \cdot \rvert$	Absolute
$\lVert \cdot \rVert$	l^2-Norm or Euclidean Norm
$(\cdot)^{\mathrm{H}}$	Complex conjugate transpose
$(\cdot)^{-1}$	Inverse of a Matrix
$\#(\cdot)$	Cardinality of $(\cdot)$
$\arg(\cdot)$	Argument of $(\cdot)$
$\exp(\cdot)$	Exponential function to base e
$\mathcal{E}(\cdot)$	Expected value of $(\cdot)$
$f(\cdot)$	Function of $(\cdot)$
$\ln(\cdot)$	Logarithm to the base e of $(\cdot)$ (natural logarithm)
$map(\mathbf{c})$	Mapping($\mathbf{c}$) on transmit vector
$\max\limits_{\mathbf{x}}\{\cdot\}$	Maximum of $\{\cdot\}$ over $\mathbf{x}$
$\min\limits_{\mathbf{x}}\{\cdot\}$	Minimum of $\{\cdot\}$ over $\mathbf{x}$
$P(\cdot)$	Probability of $(\cdot)$

Symbols

$\mathbf{0}_{n\times m}$	Null matrix of Dimension $n \times m$
a	Distance between QAM constellation symbols
b	Bit value $\in \pm 1$
$c_{m,l}$	l^{th} bit of the symbols send by m^{th} transmit antenna; $c_{m,l} \in \pm 1$
$c_{m,l}^{\text{ML}}$	ML estimate of $c_{m,l}$
$\mathbf{c}$	Vector of transmit bits of a transmission symbol vector $\mathbf{x}$;
$\mathbf{c}^{\text{ML}}$	ML estimate of $\mathbf{c}$
$\mathbf{c}_t$	t^{th} vector of transmitted bits
$\mathbf{c}_\mathbf{i}$	Part of the vector of transmitted bits from tree root to tree level i.
d	Predefined distance
e	Euler number ($\approx 2,7182818$)
E_b	Average energy per bit
E_x	Average transmit energy
$\mathbf{G}$	Filter matrix
$\mathbf{G}_{\text{MMSE}}$	MMSE Filter matrix
$\mathbf{G}_{\text{ZF}}$	ZF Filter matrix
$\mathbf{H}$	Channel matrix
$\bar{\mathbf{H}}$	Extended channel matrix for MMSE preprocessing
$\mathbf{I}_n$	Identity matrix of Dimension $n \times n$
K	Size of candidates list, $K = \#\mathcal{K}$
L	Number of bits per QAM symbol
$L(\cdot)$	A Posteriori information
$L_\text{a}(\cdot)$	A Priori information
$\mathbf{L}_\text{a}(\cdot)$	Vector of a priori information
L_{Clip}	Internal Clipping value
$L_\text{e}(\cdot)$	Extrinsic information
$L_{\max}$	Clipping value
$\mathbf{n}$	AWGN noise vector
$\tilde{\mathbf{n}}$	Vector of remaining noise after detection
M	Maximum number of iterations
N_0	Noise power
N_T	Number of transmit antennas
N_R	Number of receive antennas
Q	QAM Constellation size
$\mathbf{Q}$	Matrix of the QR decomposition
$r_{i,j}$	Element of the $\mathbf{R}$ Matrix with row i and column j
R	Search radius
R_c	Code rate

R_{Clipped}	Search radius with internal clipping
$\mathbf{R}$	Matrix of the QR decomposition
T	Size of search tuple, $T = \#\mathcal{T}$
$\mathbf{u}$	Vector of uncoded i.i.d. information bits
$\hat{\mathbf{u}}$	Vector of estimated information bits $\mathbf{u}$
$\mathbf{x}$	Sent symbol vector
$\hat{\mathbf{x}}$	Estimate of sent symbol vector
$\hat{\mathbf{x}}^{\text{ML}}$	ML estimate of the sent symbol vector
$\hat{\mathbf{x}}^{\text{SIC}}$	SIC estimate of the sent symbol vector
${y_i}'$	Preprocessed received symbol vector of the i^{th} transmit antenna
${y_i}''$	Interference reduced received symbol vector of the i^{th} transmit antenna
${y_i}'''$	y'' normalized with r_{ii}
$\mathbf{y}$	Received symbol vector
$\sharp n$	Average number of node extensions
∂	Overhead rate
Δ^{r}	Relative position
λ_0	Metric of leaf node
λ_i	Partial metric at layer i
$\lambda_{\min}$	Minimum metric λ_0
$\lambda_{\min,t}$	t^{th} minimum metric λ_0
σ^2	Ratio of noise to transmit power
σ_{n}^2	Noise variance
σ_{x}^2	Variance of send vector
$\mathcal{C}$	set of valid send vector $\mathbf{c}$
$\mathcal{K}$	Set of candidates for L-values calculation
$\mathcal{T}$	Set of leaf nodes in search tuple
$\mathcal{X}$	Set of valid send symbol vector $\mathbf{x}$

Chapter 1

Introduction

1.1 Motivation and Research Focus

To fulfil the demand for ever growing data rates, future mobile communication systems will make use of multiple-input multiple-output (MIMO) techniques to enhance spectral efficiency. If the propagation environment offers a sufficient amount of spatial diversity, the spectral efficiency achievable with MIMO systems scales linearly with the minimum of the number of transmit and receive antennas. However, this performance improvement comes at the cost of increased computational complexity in the receiver. One of the main challenge is the computationally intense task of MIMO detection use to separate the spatially multiplexed data streams. Multiple antennas in a system introduce interference on each other, making the allocation of received signal values to the most likely sent symbols a complex task. Over the years, MIMO detection algorithms for spatially-multiplexed signals have been thoroughly investigated. The optimal solution to detect spatially-multiplexed signals relies on exhaustive search over a multi-dimension constellation set, whose size grows exponentially with the number of antennas. However, the heavy computational burden of such an optimal detection is impractical for implementation. The well-known sphere detector (SD)[Poh81] effectively transforms the exhaustive search into a constrained tree search with extensive pruning of irrelevant branches, and hence is regarded as a pragmatic solution for the MIMO detection problem [Lai11].
Triggered by the tremendous gains in data rate provided by iterative processing of channel codes, the concept of iterative processing has recently propagated further to include iterations between MIMO detector and channel decoder. Soft-In Soft-Out (SISO) detectors, when concatenated with a SISO channel decoder, can significantly improve the quality of wireless transmission by performing joint, iterative data detection and channel decoding through the exchange of soft information. However, the exponential complexity of the optimal maximum a posterior probability detector rapidly becomes prohibitive. This motivates the design of suboptimal SISO detectors whose complexities are scalable with system dimensions. Complexity reduced SISO MIMO detection algorithms, e.g. list sphere detector

(LSD) [HtB03], Single tree search (STS) [SBB08] and Tuple search (TS) [MFF09], has shown to be of high accuracy but also of high complexity, resulting from the complexity of the underlying tree searches. In order to reduce this complexity, [HtB03] proposed to accomplish the iterations on a candidate list generated in the first iterations, leading to a significant performance loss and high memory requirements [MFF09].
The focus of the work presented in this thesis is to reduce the complexity of the SISO detector at minimum or even no performance loss and to enable high throughput. The presence of a soft information in iterative detection↔decoding increases the search space and hereby the computational complexity of the tree search in SISO detector. In this work it is shown how this complexity can be reduced by limiting the tree search to uncertain bits i.e. bits with low soft information. In order to demonstrate the impact of this concept on performance and complexity of a SISO detector, the TS algorithm [MvBF09] together with complexity reduction techniques of Search Sequence Determination (SSD) [MF09a] and Metric Estimation (ME) [Men10] is used. For hard-input, the complexity of TS algorithm is greatly reduced by the geometrical approach of SSD. In case of iterative detection↔decoding, the soft information from channel decoder destroys the geometrical properties of constellation and as a result SSD leads to wrong sequence of nodes for tree enumeration. The proposed concept of limiting the tree search to uncertain bits, besides the intended reduction in complexity of the tree search, will also help to correct the enumeration sequence determined by SSD. To increase throughput of the SISO detection, this work has introduced parallel processing in the TS algorithm. A case study on MIMO detection in 3GPP Long Term Evolution (LTE) [3GP06] is also a part of this work. The final contribution of this work is the VLSI implementation of the Processor model for MIMO detection.

1.2 Outline

Chapter 2 describes the used system model and basic considerations of the MIMO detection. It also formulates the problem of MIMO detection addressed in this work.

Chapter 3 introduces a generic framework for tree search based detection. Sphere detection algorithm in the context of iterative detection and decoding is described. It is followed by description of the algorithms related to this work. Several complexity reduction techniques are also mentioned in this chapter.

Chapter 4 presents a noval method for reducing the complexity of MIMO detection in joint iterative detection and decoding. The chapter starts by describing the proposed method in detail. It then explains how the proposed method helps in finding the search sequence for tree enumeration in SISO detection. Simulation results are presented in the last part of the chapter.

Chapter 5 introduces parallel processing in TS algorithm to speed up the detection and hence increasing the throughput of the algorithm. The chapter first highlights the problems of the TS algorithm due to sequential node processing. It then describes the methods of parallel processing in detail. Finally this chapter provides the simulation results and efficiency analysis of the the resulting algorithm.

Chapter 6 is a case study on MIMO detection in 3GPP LTE system. Simulations results of the several detection algorithms for different LTE channel models are provided in this chapter.

Chapter 7 provides an overview of the processor model for TS algorithm. The chapter also presents the synthesis results for VLSI implementation of the processor model and its comparison to state of the art implementations.

Chapter 8 concludes the thesis with a summary of the main results and an outlook.

Chapter 2

Fundamentals

This chapter is a brief overview of the used system model, basic considerations of the MIMO detection and formulation of the problem addressed in this work. The details of development of the tree search methods and techniques that will enable the implementation of efficient MIMO detectors are well known and are therefore referred to standard literature.

2.1 System Model

For comparability of the results with other publications and continuation of works in [Men10] and [Zim07], a MIMO system model with a QAM (Quadrature Amplitude Modulation) constellation is considered in this thesis. This system model, shown in Fig.2.1, will serve as basis for the subsequent discussions to ensure that the results are applicable to a wide range of communication scenarios and to provide a common base for the comparison of different detection algorithms. The propagation environment between transmitter and receiver is assumed to be non line of sight with sufficient scattering to provide large number of independent transmission paths. Further, to enable low complexity equalization at the receiver, a frequency non selective narrowband channel is considered in this work. Since broadband channels can be reduced to narrowband channels with proper modulation techniques such as Orthogonal Frequency Division Multiplexing (OFDM), MIMO detection techniques developed for transmission over narrowband channels can be reused for broadband systems. The task of the detector here is enabling a transmission even with a weak or noisy received signals close to the theoretical performance limit with highest possible throughput. Many research works have been carried out for the design of hard output MIMO detector. The performance of MIMO detector can be significantly improved by iterative detection↔decoding. However, its complexity limits the overall throughput. Therefore, the work carried out in this thesis is focused on the development and analysis of efficient MIMO detector for iterative detection↔decoding.

The system model under consideration is an $N_\mathrm{T} \times N_\mathrm{R}$ MIMO system based on a bit-interleaved coded modulation (BICM) [CTB98] transmission strategy with N_T transmit

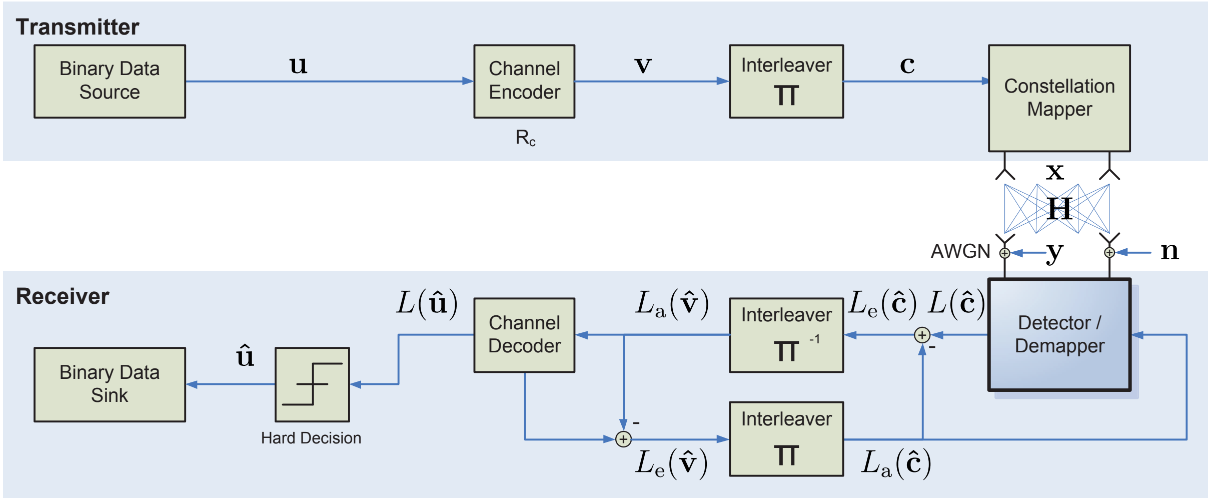

Figure 2.1: System model with BICM transmitter and iterative receiver.

and N_R receive antennas. A vector $\mathbf{u}$ of independent and identically distributed (i.i.d.) information bits is encoded by an outer channel code with rate R_c. The coded vector $\mathbf{v}$ is bit-interleaved and portioned into blocks $\mathbf{c}$ of $N_\mathrm{T}{\cdot}L$ bits, where L denotes the number of bits per transmit symbol. For the transmission, the corresponding bits $\mathbf{c} \in \mathcal{C}$, covered in the set of permitted bit vectors, are mapped (e.g. gray mapping) onto complex constellation symbols $\mathbf{x}(\mathbf{c}) = [x_0, ...x_{N_\mathrm{T}-1}]^T = map(\mathbf{c}) \in \mathcal{X}$, the set of valid transmit symbols with cardinality $\#\mathcal{X} = \#\mathcal{C} = 2^L$. The transmit power is normalized such that $\mathcal{E}\{\mathbf{x}\mathbf{x}^\mathrm{H}\} = E_\mathrm{x}/N_\mathrm{T}\mathbf{I}_{N_\mathrm{T}}$, with E_x being the average transmit power of $\mathbf{x}$ at the transmitter. On behalf of the transmission, a flat fading channel and an additive white Gaussian noise (AWGN) vector $\mathbf{n} \in \mathbb{C}^{\mathbf{N_\mathrm{R}\times 1}}$ with complex components of zero mean i.i.d. gaussian random variables is considered at the receiver. The noise power density is $N_0/2$ per real dimension ($\mathcal{E}\{\mathbf{n}\mathbf{n}^\mathrm{H}\} = N_0\mathbf{I}_{N_\mathrm{R}}$). The considered passive channel is represented by $\mathbf{H} \in \mathbb{C}^{N_\mathrm{T}\times N_\mathrm{R}}$ with entries of a zero mean i.i.d. gaussian random process of variance 1 and is assumed to be perfectly known at the receiver. The received signal $\mathbf{y}$ is therefore given by:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2.1}$$

with the following multidimensional Gaussian distribution of the complex received signal (as given e.g. in [W0̈5]):

$$P(\mathbf{y}|\mathbf{x}) = \frac{1}{(\pi N_0)^{N_\mathrm{R}}} e^{-\frac{\|\mathbf{y}-\mathbf{H}\hat{\mathbf{x}}\|^2}{N_0}} \tag{2.2}$$

The number of transmitted information bits per vector symbol is $N_T.L.R_c$. With the average received energy per vector symbol given by $E_x.N_R$, the signal-to-noise-ratio at the receiver applied to the energy of one information bit, E_b, can be stated as:

$$\frac{E_\mathrm{b}}{N_0} = \frac{1}{N_T.L.R_c}\frac{(E_\mathrm{x}/N_\mathrm{T})(N_T N_R)}{N_0} = \frac{E_\mathrm{x}}{N_0}\frac{N_\mathrm{R}}{N_\mathrm{T}}\frac{1}{L.R_\mathrm{c}}$$

In order to ensure comparability of the results, a simulation setup equivalent to the one used in e.g. [HtB03, ZF06] [Men10] [Zim07] is considered. The simulations are carried out for a rate 1/2 PCCC (Parallel Concatenated Convolutional Code, a so called Turbo Code [BGT93, HOP96]) with two constituent convolutional encoders linked by an interleaver and with $(7_R, 5)$ [1] convolutional code polynomial. The length of an information block is 9216 bits (including tail bits). For the transmission, 64 QAM, Gray Mapping and a 4×4 MIMO system ($N_R = N_T = 4$) was chosen. On the receiver the detection of the transmitted bits is carried out by a complex valued SISO MIMO detector with and without detector↔decoder iterations according to the turbo principle [Hag02]. The vector $\hat{\mathbf{c}}$, which is an estimate of the detected bits, is deinterleaved and passed to the channel decoder as a vector $\hat{\mathbf{v}}$. The interleaver at the transmitter and receiver is a random interleaver and it interleaves over the whole length of the codeword. The channel decoder used is a turbo decoder [RVH95] based on the BCJR algorithm [BGT93] with 8 internal iterations. $\hat{\mathbf{u}}$ is a vector of the estimated bits at the output of the channel decoder. Further details and analysis relevant to the used system model can be found in [HtB03], [Zim07] and [Men10].

2.2 MIMO Detection

The task of the MIMO detector in the described system model is to find an estimate $\hat{\mathbf{x}}$ of the transmitted signal vector $\mathbf{x}$, given the received signal $\mathbf{y}$ according to equation 2.1. The channel matrix $\mathbf{H}$ is assumed to be perfectly known and the noise vector $\mathbf{n}$ is unknown. There are many approaches for the detection of the transmitted signal $\mathbf{x}$. Some of the most common approaches are mentioned in the following.

2.2.1 Maximum-Likelihood Detection

If we are not interested in soft output and no a priori knowledge from the decoder is available, the transmitted signal can be detected by maximum likelihood (ML) detection [vE76]:

$$\begin{aligned} \hat{\mathbf{x}}^{\mathrm{ML}} &= \arg\min_{\mathbf{x}\in\mathcal{X}} \left\{ P\left(\mathbf{x}|\mathbf{y}\right) \right\} && (2.3) \\ &= \arg\min_{\mathbf{x}\in\mathcal{X}} \left\{ \|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2 \right\} && (2.4) \end{aligned}$$

A straight forward approach to solve equation (2.4) is an exhaustive search over the entire set of possible vector symbols $\mathbf{x} \in \mathcal{X}$ to find $\hat{\mathbf{x}}^{ML}$. It provides an optimum detection performance by minimizing bit error rate (BER). Unfortunately the computational complexity is NP hard as it increases exponentially with number of transmit antennas N_T and QAM constellation size.

2.2.2 Linear Detection

Linear detectors are attractive whenever some performance degradation can be accepted in order to achieve very low receiver complexity. The linear detector applies a linear filter

[1] The subscript R indicates the recursive or feedback generator polynomial.

matrix to the received signal to compensate the effect of the channel [WFGV98]:

$$\hat{\mathbf{x}} = \mathbf{G}\mathbf{y} = \mathbf{G}\mathbf{H}\mathbf{x} + \mathbf{G}\mathbf{n} = \mathbf{G}\mathbf{H}\mathbf{x} + \tilde{\mathbf{n}}.$$

Where $\mathbf{G} \in \mathbb{C}^{\mathbf{N}_{\mathrm{R}} \times \mathbf{N}_{\mathrm{T}}}$ is a linear filter and $\tilde{\mathbf{n}}$ is the the resulting noise vector. There are two types of linear detectors.

Zero Forcing (ZF)

The ZF linear detector finds an estimate of the transmitted signal $\mathbf{x}$ by solving equation (2.1) regardless of the noise:

$$\hat{\mathbf{x}} = \mathbf{G}_{\mathbf{ZF}}\mathbf{y}$$

Where $\mathbf{G}_{\mathbf{ZF}}$ is a linear filter matrix obtained by taking Pseudo-Inverse [SB07] of the channel matrix as follows

$$\mathbf{G}_{\mathrm{ZF}} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{H}}$$

Despite its simplicity, this approach suffers from the noise enhancement problem [Zim07].

Minimum Mean Squared Error (MMSE)

In MMSE detection the influence of receiver noise is considered in the design of filter matrix to overcome the noise enhancement problem of ZF detection. In this case the matrix filter is defined as

$$\mathbf{G}_{\mathrm{MMSE}} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} - \sigma^2\mathbf{I}_{N_{\mathrm{T}}})^{-1}\mathbf{H}^{\mathrm{H}}.$$

With uniform distribution of transmit power per antenna $E_{\mathrm{x}}/N_{\mathrm{T}}$ and noise power N_0, the σ^2 for MMSE detection is given by

$$\sigma^2 = \frac{N_0}{E_{\mathrm{x}}/N_{\mathrm{T}}} = \frac{N_{\mathrm{T}}N_0}{E_{\mathrm{x}}} \tag{2.5}$$

2.3 Iterative Detection and Decoding

For the used system model with coded transmission, it is suboptimal for the MIMO detector and channel decoder to operate separately and only on individual vectors of the received signal. Optimal system performance is achieved only if the detector makes decisions jointly on all the vectors using a priori information provided by the channel decoder and the channel decoder makes decisions using likelihood information on all the vectors obtained from the MIMO detector. Therefore, the application of the Turbo principle [HOP96] for iterative detection↔decoding is considered. As shown in Fig. 2.1 the receiver consists of the serial concatenation of an inner MIMO detector and an outer channel decoder. Both modules accept and generate soft information on the bits of the transmitted codeword $\mathbf{c}$. The detector exploits its knowledge of the received signal, the channel state information and the a priori information $L_{\mathrm{a}}(\hat{\mathbf{c}})$ provided by the decoder to generate the a posteriori information $L(\hat{\mathbf{c}})$. The extrinsic information $L_{\mathrm{e}}(\hat{\mathbf{c}})$, obtained by subtracting a priori information $L_{\mathrm{a}}(\hat{\mathbf{c}})$ from a posteriori information $L(\hat{\mathbf{c}})$, is deinterleaved to become the a priori input $L_{\mathrm{a}}(\hat{\mathbf{v}})$

to the channel decoder. The channel decoder calculates the a posteriori information $L(\hat{\mathbf{u}})$ on the outer coded bits using BCJR algorithm [BGT93]. The extrinsic information $L_{\mathrm{e}}(\hat{\mathbf{v}})$ is then interleaved and passed on as a priori knowledge $L_{\mathrm{a}}(\hat{\mathbf{c}})$ to the inner detector, thus completing an iteration. Conventionally used Log-Likelihood Ratios (L-values) provide a convenient notation to describe the soft-output in iterative detection↔decoding. The L-value of the bit $\hat{c}_{m,l}$ is defined as follow

$$L\left(\hat{c}_{m,l}|\mathbf{y}\right) = \ln\left(\frac{P\left(\hat{c}_{m,l}=+1|\mathbf{y}\right)}{P\left(\hat{c}_{m,l}=-1|\mathbf{y}\right)}\right) \tag{2.6}$$

As shown in Fig. 2.1, each processing module in iterative detection↔decoding deals with three different types of information: the a priori information received from the other module, the a posteriori information generated by this module, and the extrinsic information sent back to the other module. The L-values of the a priori, a posteriori and extrinsic information on a bit $\hat{c}_{m,l}$ are indicated by $L_{\mathrm{a}}\left(\hat{c}_{m,l}\right)$, $L\left(\hat{c}_{m,l}|\mathbf{y}\right)$ and $L_{\mathrm{e}}\left(\hat{c}_{m,l}|\mathbf{y}\right)$ respectively. Their relation can be expressed as

$$\underbrace{\ln\frac{P\left(\hat{c}_{m,l}=+1|\mathbf{y}\right)}{P\left(\hat{c}_{m,l}=-1|\mathbf{y}\right)}}_{\text{A-Posteriori Information}} = \underbrace{\ln\frac{P\left(\hat{c}_{m,l}=+1\right)}{P\left(\hat{c}_{m,l}=-1\right)}}_{\text{A-Priori Information}} + \underbrace{\ln\frac{p\left(\mathbf{y}|\hat{c}_{m,l}=+1\right)}{p\left(\mathbf{y}|\hat{c}_{m,l}=-1\right)}}_{\text{Extrinsic Information}}$$

$$L\left(\hat{c}_{m,l}|\mathbf{y}\right) = L_{\mathrm{a}}\left(\hat{c}_{m,l}\right) + L_{\mathrm{e}}\left(\hat{c}_{m,l}|\mathbf{y}\right)$$

The following subsections give an overview of the basic approaches to calculate the a posteriori L-values of the MIMO detector.

2.3.1 A Posteriori Probability Detection

The a posteriori L-values at the output of the MIMO detector can be calculated by commonly known form of equation (2.6):

$$L\left(\hat{c}_{m,l}|\mathbf{y}\right) = L_{\mathrm{a}}\left(\hat{c}_{m,l}\right) + ln\frac{\sum\limits_{\mathbf{x}\in\mathcal{X}^{+1}_{m,l}}\exp\left(-\frac{1}{N_0}\|\mathbf{y}-\mathbf{H}\hat{\mathbf{x}}\|^2+\frac{1}{2}\sum\limits_{i,j\neq m,l}\hat{c}_{i,j}(\mathbf{x})L_{\mathrm{a}}\left(\hat{c}_{i,j}\right)\right)}{\sum\limits_{\mathbf{x}\in\mathcal{X}^{-1}_{m,l}}\exp\left(-\frac{1}{N_0}\|\mathbf{y}-\mathbf{H}\hat{\mathbf{x}}\|^2+\frac{1}{2}\sum\limits_{i,j\neq m,l}\hat{c}_{i,j}(\mathbf{x})L_{\mathrm{a}}\left(\hat{c}_{i,j}\right)\right)}. \tag{2.7}$$

Where $\hat{c}_{m,l} = \pm 1$ represents the l-th bit of the symbol sent by the m-th antenna. The derivation of equation (2.7) from equation (2.6) is detailed in Appendix A. The optimal detection strategy is to evaluate equation (2.7) by a brute-force approach and is referred as A Posteriori Probability (APP) detection.

2.3.2 MaxLogAPP Detection

The computational effort in the APP detection can be greatly reduced by using the Jacobian logarithm [RVH95] and applying the so called max-log-approximation (MaxLogAPP). The a posteriori L-values become

$$L\left(\hat{c}_{m,l}|\mathbf{y}\right) \;\approx\; -\frac{1}{N_0}\min_{\hat{\mathbf{c}}|\hat{c}_{m,l}=+1}\{\lambda_0\} + \frac{1}{N_0}\min_{\hat{\mathbf{c}}|\hat{c}_{m,l}=-1}\{\lambda_0\}, \tag{2.8}$$

$$\lambda_0 = \|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}(\hat{\mathbf{c}})\|^2 - \frac{N_0}{2}\sum_{i=0}^{N_\mathrm{T}-1}\sum_{j=1}^{L}\hat{c}_{i,j}L_\mathrm{a}(\hat{c}_{i,j}), \tag{2.9}$$

where λ_0 represents the distance metric for a vector of received symbols $\mathbf{y}$, a given $\hat{\mathbf{c}}$ and the a priori knowledge $\mathbf{L_a}$. $\hat{\mathbf{x}}$ corresponds to a possible transmission symbol. As a consequence, beside the most properly sent symbol $\arg\min_{\hat{\mathbf{x}}(\hat{\mathbf{c}})|\hat{\mathbf{c}}\in\mathcal{C}}\{\lambda_0\}$ - the detection hypothesis - and its corresponding metric $\lambda_0(\hat{\mathbf{c}}^{\mathrm{ML}})$, the detector has to determine also the counter-hypotheses $\arg\min_{\hat{\mathbf{x}}(\hat{\mathbf{c}})|\hat{\mathbf{c}}\in\mathcal{C},\hat{c}_{m,l}\neq\hat{c}_{m,l}^{\mathrm{ML}}}\{\lambda_0\}$ with their metrics for each bit.
Derivation of equations 2.8 and 2.9 can found in Appendix A.

2.3.3 Tree Search Based MIMO Detection

The basic aim of calculating the L-values in equation (2.8) is to determine the most likely sent symbols $\mathbf{x}(\mathbf{c})$, with $\hat{\mathbf{c}} = \arg\min_{\hat{\mathbf{c}}|\hat{c}_{m,l}=\pm 1}\{\lambda_0\}$. A common approach to simplify the detection of these symbols is to transform the detection problem into a tree search problem using QR decomposition of the channel matrix $\mathbf{H}$ as described in section 3.1. An appropriate tree search methodology determines the most likely sent symbols without analyzing all possible sent symbols. It is this property of the tree search methods which makes them interesting for implementation of an efficient MIMO detection and forms the basis of this work. A detailed description of tree search based MIMO detection is given in the following chapters.

2.3.4 Successive Interference Cancellation

Successive interference cancellation (SIC) [Zim07] is a simple approach to find the symbols for equation (2.8) with minimum computational effort. In this case detection takes place successively layer by layer estimating the transmitted symbols sequentially. The interference from already detected layers is removed from the received signal before detecting the next layer. This continues until all symbols $\hat{\mathbf{x}}^{\mathrm{SIC}}$ have been detected. There are a number of different approaches for the implementation of the SIC detection. As part of this work, it is included as a reference for tree search based detection and is therefore realized with similar processing steps as the tree search. Further details can be found in standard literature.

2.4 Conclusion

In this chapter an overview of the used system model and MIMO detection methods together with their performance and complexity is provided. For ML detection the complexity grows exponentially with the number of transmit antennas and even for flat fading channels it is prohibitively complex at higher modulation order. On the other hand, low complexity linear detectors are less robust and suffer from noise enhancement. Iterative detection↔decoding offers the best complexity-performance tradeoff and is therefore used for MIMO detection in this work. The complexity of calculating soft-output in iterative detection↔decoding can be greatly reduced using MaxLogAPP.