

Ines Riedel

N-Fold Sectorization in the Light of Intra-Site  
Coordinated Multi-Point Transmission



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**N-Fold Sectorization in the Light of Intra-Site  
Coordinated Multi-Point Transmission**

**Ines Riedel**

von der Fakultät Elektrotechnik und Informationstechnik  
der Technischen Universität Dresden

zur Erlangung des akademischen Grades

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# Abstract

Mobile data communication has become ubiquitous during the last few years. To meet the exponential growth of mobile data traffic, mobile network operators face the challenge of significantly increasing their network capacity. Furthermore, the spatial distribution of the available user data rates has to become more homogeneous to improve the user experience. An increase in the number of base stations and coordinated multi-point transmit and receive techniques are among the most promising concepts to accommodate both objectives in sectorized cellular networks. This thesis focuses on the degrees-of-freedom in the design of sectorized cellular networks using so-called intra-site coordinated multi-point transmission.

In order to investigate the degrees-of-freedom available, a generalized concept of modeling such networks is proposed that can be used to analyze networks with arbitrary extents of sectorization and overlapping coverage areas. Furthermore, an advanced three-dimensional base station antenna model is developed that allows tuning major antenna radiation characteristics, and also incorporates an antenna gain formulation. To assess the potential performance of sectorized cellular networks, transmission concepts with and without intra-site base station cooperation are selected for a detailed analysis.

Based on this, the impact of the extent of sectorization and the impact of major antenna radiation characteristics are evaluated through a system level analysis. The sensitivity to non-full load and the impact of the degree of cooperation are analyzed as well. Moreover, the results are extended to consider multi-antenna base stations. Thus, it is shown that higher extents of sectorization can, indeed, improve the network performance. However, for conventional non-cooperative transmission, this gain has to be compensated by a user performance degradation. One of the key results is that the application of joint Wiener filtering at 6-sector sites with  $35^\circ$  antennas already achieves 77% of the average network throughput gain that can be obtained while switching from these 6-sector to 12-sector sites with  $17.5^\circ$  antennas.

The last part of this thesis discusses practical implications of the theoretical results obtained. One of these is the fact that intra-site cooperation renders  $70^\circ$  antennas, i.e., typical antennas for 3-sector sites, attractive even for cellular network designs with higher extents of sectorization.

In general, it is found that intra-site cooperation in conjunction with higher extents of sectorization is a suitable means to improve network and user throughput as well as the spatial homogeneity of user data rates.





# Zusammenfassung

Mobile Datenkommunikation ist in den letzten Jahren allgegenwärtig geworden. Um dem exponentiellen Wachstum des mobilen Datenverkehrs gerecht zu werden, müssen die Mobilfunkbetreiber ihre Netzkapazität signifikant zu erhöhen. Um darüber hinaus das Nutzererlebnis zu verbessern, muss auch die flächige Verteilung des erreichbaren Datendurchsatzes weiter homogenisiert werden. Eine Erhöhung der Anzahl der Basisstationen sowie kooperative verteilte Sende- und Empfangsverfahren gehören zu den vielversprechendsten Ansätzen, beiden Zielsetzungen gerecht zu werden. Die vorliegende Arbeit untersucht die Gestaltungsmöglichkeiten sektorisierter zellulärer Netzwerke angesichts einer möglichen Kooperation von Basisstationen innerhalb eines gemeinsamen Antennenstandortes.

Um die Freiheitsgrade der Netzwerkgestaltung untersuchen zu können, wird ein verallgemeinerter Ansatz zur Modellierung dieser Netzwerke vorgeschlagen. Dieser Ansatz ermöglicht die Untersuchung von Netzwerken mit einem beliebigen Grad der Sektorisierung und überlappenden Abdeckungsgebieten. Darüber hinaus wird ein erweitertes dreidimensionales Basisstations-Antennenmodell entwickelt, das die Variation wichtiger Antennencharakteristika ermöglicht und den Antennengewinn berücksichtigt. Um die Leistungsfähigkeit sektorisierter zellulärer Netzwerke bewerten zu können, werden Übertragungskonzepte mit und ohne Kooperation von Basisstationen für eine detaillierte Analyse ausgewählt.

Basierend darauf, werden der Grad der Sektorisierung sowie wichtige Antennencharakteristika hinsichtlich ihrer Auswirkungen auf Systemebene analysiert. Des Weiteren werden die Sensitivität gegenüber einer nicht vollen Systemauslastung und der Einfluss des Kooperationsgrades diskutiert. Die gewonnenen Ergebnisse werden auf Systeme mit Mehrantennen-Basisstationen erweitert. Es wird gezeigt, dass ein erhöhter Grad der Sektorisierung tatsächlich den Netzwerkdurchsatz erhöhen kann. Dieser Gewinn geht jedoch im Falle nichtkooperativer Übertragung mit einer Verschlechterung des Nutzerdurchsatzes einher. Eine der Hauptkenntnisse dieser Arbeit ist, dass die gemeinsame Wiener Filterung an einem 6-Sektor Standort mit  $35^\circ$  Antennen bereits 77% der Steigerung des mittleren Netzwerkdurchsatzes erreicht, die durch den konventionellen Umbau zu einem 12-Sektor Standort mit  $17.5^\circ$  Antennen möglich ist.

Der letzte Abschnitt dieser Arbeit diskutiert praktische Schlussfolgerungen der theoretischen Betrachtungen. Eine dieser Schlussfolgerungen ist, dass die Kooperation von Basisstationen innerhalb eines gemeinsamen Antennenstandortes die Verwendung von  $70^\circ$  Antennen, also typischen Antennen von 3-Sektor Standorten, auch für Antennenstandorte mit einem höheren Grad der Sektorisierung attraktiv macht.

Insgesamt zeigt diese Arbeit, dass die Kooperation von Basisstationen innerhalb eines gemeinsamen Antennenstandortes in Verbindung mit einem erhöhten Grad der Sektorisierung eine geeignete Möglichkeit ist, sowohl den Netzwerk- und Nutzerdurchsatz zu erhöhen, als auch die Homogenität der erreichbaren Nutzerdatenrate in der Fläche zu verbessern.

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# Acronyms

<b>2G</b>	second generation (GSM)
<b>3G</b>	third generation (UMTS)
<b>3GPP</b>	3rd Generation Partnership Project
<b>4G</b>	fourth generation (LTE-Advanced)
<b>BC</b>	broadcast channel
<b>BS</b>	base station
<b>CAPEX</b>	capital expenditures
<b>CDMA</b>	code division multiple access
<b>CoMP</b>	coordinated multi-point
<b>CPRI</b>	common public radio interface
<b>DC</b>	direct current
<b>DPC</b>	dirty paper coding
<b>CSI</b>	channel state information
<b>DL</b>	downlink
<b>GSM</b>	Global System for Mobile Communications (second generation, 2G)
<b>HPBW</b>	half-power beamwidth (3 dB beamwidth, beamwidth)
<b>IEEE</b>	Institute of Electrical and Electronics Engineers
<b>ISC</b>	intra-site coordinated multi-point
<b>ISC-DPC</b>	intra-site coordinated multi-point dirty paper coding
<b>ISC-WF</b>	intra-site coordinated multi-point Wiener filtering
<b>LOS</b>	line-of-sight
<b>LTE</b>	Long-Term Evolution
<b>MAC</b>	multiple-access channel
<b>MIMO</b>	multiple-input multiple-output
<b>MMSE</b>	minimum mean squared error
<b>MRT</b>	maximum ratio transmission
<b>MS</b>	mobile station
<b>NCT</b>	non-cooperative transmission
<b>NLOS</b>	non line-of-sight
<b>OBSAI</b>	open base station architecture initiative
<b>OFDMA</b>	orthogonal frequency division multiple access
<b>OPEX</b>	operational expenditures
<b>PIFA</b>	planar inverted F antenna

<b>QoE</b>	quality of experience
<b>RAN</b>	radio access network
<b>RAT</b>	radio access technology
<b>RF</b>	radio frequency
<b>RMa</b>	rural macro (3GPP scenario for path loss modeling)
<b>RRH</b>	remote radio head
<b>SC</b>	subcarrier
<b>SNR</b>	signal-to-noise ratio
<b>SINR</b>	signal-to-interference-and-noise ratio
<b>SIR</b>	signal-to-interference ratio
<b>THP</b>	Tomlinson-Harashima precoding
<b>TTI</b>	transmission time interval
<b>UE</b>	user equipment
<b>UHF</b>	ultra-high frequency
<b>UMa</b>	urban macro (3GPP scenario for path loss modeling)
<b>UMTS</b>	Universal Mobile Telecommunications System (third generation, 3G)
<b>VDSL2</b>	Very-High-Speed Digital Subscriber Line 2
<b>WCDMA</b>	wideband code division multiple access
<b>WF</b>	Wiener filtering
<b>XG-PON</b>	10-Gigabit-capable passive optical networks



# Symbols and Notation

$A$	Antenna radiation pattern (here: normalized radiation intensity)
$A_{\text{BW}}$	Backward attenuation in linear scale
${}^s\mathcal{A}_n$	Area served by BS $n$ and associated with sector $n$ of antenna site $s$
${}^s\mathcal{B}$	Index set comprising all active BSs of antenna site $s$
$c_0$	Speed of light
${}^s\mathcal{C}_n$	Area covered by BS $n$ and associated with sector $n$ of antenna site $s$
$D$	Inter-site distance
$\mathbf{d}$	Data vector ( $ \mathcal{K}  \times 1$ )
$\det \cdot$	Determinant
$E\{\cdot\}$	Expectation
$\text{erf}(\cdot)$	Error function ( $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ )
$F_{\text{reuse}}$	Reuse factor
$f_c$	Carrier frequency
$G$	Antenna gain, in dBi if represented in logarithmic scale
$[\mathbf{H}]_{k,n}$	Entry in the $k$ th row and $n$ th column of matrix $\mathbf{H}$
$h_{k,n}$	Entry in the $k$ th row and $n$ th column of matrix $\mathbf{H}$
$I$	Number of active interfering BSs
$\mathcal{I}$	Index set comprising all BS of all interfering antenna sites
$\mathbf{I}_N$	$N \times N$ identity matrix
${}^sK$	Number of MSs served by antenna site $s$
${}^s\mathcal{K}$	Index set comprising all MSs served by antenna site $s$
$N$	Extent of sectorization (number of sectors per antenna site, number of BSs per antenna site)
$N_{\text{BS}}$	Number of antennas per BS
$\mathcal{N}$	Index set comprising all BSs of an arbitrary antenna site
$P_{\text{Rx,min}}$	Receiver sensitivity
${}^sP_{\text{Rx},n}(x)$	Received power at location $x$ from BS $n$ of antenna site $s$
$P_{\text{site}}^{\text{max}}$	Maximum allowed transmit power per antenna site
${}^sP$	Transmit power of antenna site $s$
$\mathbf{Q}_k$	Transmit covariance matrix MS $k$
$S$	Number of antenna sites
SINR	Signal-to-interference-and-noise ratio
SINR <sub>req</sub>	Required signal-to-interference-and-noise ratio

---

$\text{tr}\{\cdot\}$	Trace operation
$\mathbf{t}$	Transmit vector of central antenna site ( $ \mathcal{B}  \cdot N_{\text{BS}} \times 1$ )
${}^s\mathbf{t}$	Transmit vector of interfering antenna site $s$ ( $ \mathcal{B}  \cdot N_{\text{BS}} \times 1$ )
$U$	Radiation intensity in watt per unit solid angle
$W_{\text{coh}}$	Coherence bandwidth
$X_{[\log]}$	Logarithmic scale value of $X$ ( $X_{[\log]} = 10 \cdot \log_{10} X$ dB)
$ X $	Absolute value of $X$
$ \mathcal{X} $	Cardinality of index set $\mathcal{X}$
$\ \mathbf{x}\ $	Euclidean vector norm ( $\ \mathbf{x}\  = \sqrt{\text{tr}\{\mathbf{x} \cdot \mathbf{x}^\dagger\}}$ )
$\Delta f$	Subcarrier spacing
$\vartheta_{3\text{dB}}$	Elevation half-power beamwidth in degrees
$\vartheta_n^{\text{BS}}$	Elevation boresight direction of base station antennas serving sector $n$ in degrees
$\vartheta_{\text{tilt}}$	Downtilt of base station antenna in degrees
$\Phi_{\mathbf{xx}}$	Covariance matrix of vector $\mathbf{x}$ ( $\Phi_{\mathbf{xx}} = E\{\mathbf{xx}^\dagger\}$ )
$\varphi_{3\text{dB}}$	Azimuthal half-power beamwidth in degrees
$\varphi_n^{\text{BS}}$	Azimuthal boresight direction of base station antennas serving sector $n$ in degrees

# Chapter 1

## Introduction

To motivate this thesis, this chapter discusses challenges for mobile cellular networks and provides an overview on related state-of-the-art approaches. After describing the focus of this thesis, related work is discussed, as well. Thereafter, the outline of this thesis including an overview of the contributions is given. Finally, the notation applied is introduced.

### 1.1 Challenges for Mobile Cellular Networks

Until 2017, analysts predict a 16-fold increase of the total global mobile data traffic and a 7-fold increase of the average mobile network connection speed over 2012 [Cis13]. To meet this exponential growth, mobile network operators face the challenge to significantly increase their network capacity. Even with an anticipated offload to fixed networks of 46%, a challenging 13-fold increase of the global mobile data traffic remains [Cis13].

Furthermore, operators do not only aim at improved average performance. A second major objective is an improvement of the quality of experience (QoE), sometimes also called quality of user experience. This subjective measure reflects the satisfaction of a user and depends on factors such as service availability, service quality and costs. In general, such a subjective measure is difficult to quantify. However, in mobile cellular networks it is reasonable to assume that a more homogeneous performance in terms of a more homogeneous spatial distribution of the data rate available to the network users will be advantageous with respect to the QoE.

Considering these two objectives and the relative price drop for mobile cellular services from 2008 to 2011 of 37% [Int11], operators face huge challenges. They have to significantly increase the network capacity as well as to improve the homogeneity of the achievable user data rates and at the same time to strictly limit the costs.

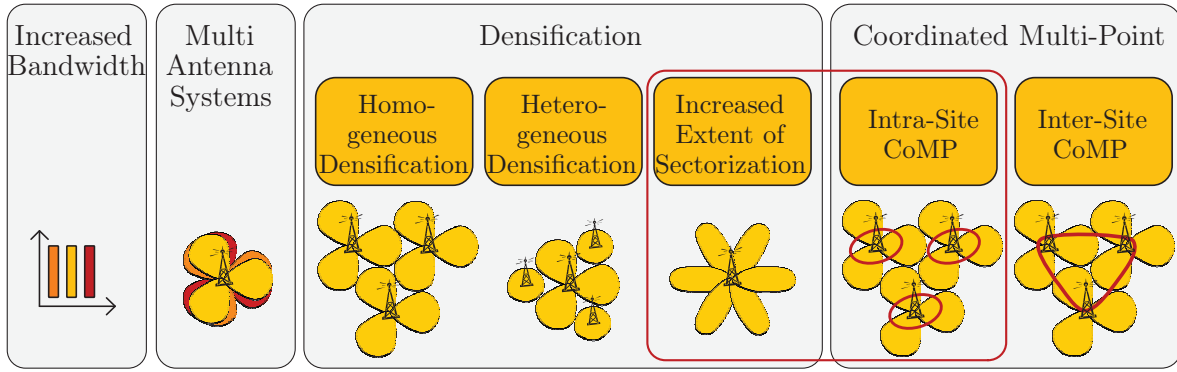


Figure 1.1: State-of-the-Art Approaches.

## 1.2 State-of-the-Art Approaches

Fig. 1.1 provides an overview of state-of-the-art approaches to cope with the discussed challenges for mobile cellular networks. In the following, we discuss these approaches.

### Increasing the Deployed Bandwidth

The most obvious option to increase the network capacity is to deploy more bandwidth by allocation of additional radio frequencies for transmission. However, the dependency of the propagation characteristics on the carrier frequency limits the technically attractive range of frequencies. Considering this, the digital dividend in the ultra-high frequency (UHF) band resulting from switching from analog to digital broadcasting is highly attractive. Consequently, operators adopted the approach and have been competing for the available spectrum in the 700 MHz and 800 MHz frequency bands during the auctions of the last years [BDW<sup>+</sup>12]. A further approach to exploit available non-contiguous spectrum fragments is spectrum aggregation [CCZY09]. Nonetheless, as the network capacity increases only linearly with the bandwidth, further advancements are required. Moreover, this approach does not alleviate the user experience problem.

### Multi-Antenna Systems

Theoretically, the throughput of multi-antenna systems may increase linearly with the minimum number of transmit and receive antennas [Tel99]. Thus, the deployment of multiple antennas at the base stations and mobile stations is an attractive further option to increase the network capacity. Various field trials proved this concept, e.g. for  $3 \times 2$  and  $4 \times 4$  systems as presented in [STT<sup>+</sup>02] and [CML<sup>+</sup>06], respectively. Consequently, so-called multiple-input multiple-output (MIMO) technologies found their way into the standardization and deployment of Long-Term Evolution (LTE) [3GP06] and LTE-Advanced [LLL<sup>+</sup>10, ETS13] cellular networks. However, an increase of the number of antennas at the base station may increase the costly rental fees for antenna sites and may aggravate the problem of the social acceptance of antenna sites. Moreover, the desired design, in particular the physical size, and costs of handsets limit the practical number

of antennas per mobile station. Consequently, practical limitations of the number of antennas bound the achievable capacity augmentation. Nevertheless, an increased number of transmit and receive antennas can also be used to exploit diversity for increasing the reliability. Thus, multi-antenna systems have the potential to additionally improve the homogeneity of the user performance.

### Densification of Base Stations

A further option to increase the network capacity is a densification of base stations in the radio access network. One way to do so is an increased number of base stations (BSs) and their respective antenna sites per area. Considering **homogeneous cellular networks**, where all cells are approximately of the same size, Liang et al. have shown in [LGF<sup>+</sup>08] that a densification of base stations can increase the spectral efficiency. However, it has also been concluded that a coordination of base stations may outperform this densification depending on the chosen transmission concept.

Another way to densify the radio access network is to supplement conventional macro BSs by micro, pico or femto BSs which are additionally deployed over the area. Commonly, such networks are referred to as **heterogeneous cellular networks** [GMR<sup>+</sup>12]. Current work in this area focuses mainly on densification as a means to increase the energy efficiency [RFF09, KFF11] and the system throughput [RF10]. However, as additional small antenna sites increase the achievable throughput in a relatively well-defined small coverage area, they can also be applied both to specifically accommodate non-uniform traffic demands and to enhance the homogeneity of the user performance by deploying these sites at previously disadvantaged locations such as cell edges. Thus, additional BSs of different types may increase the network capacity and may improve the homogeneity of the user performance. However, additional costs and effort for the construction and maintenance of extra sites remain remarkable.

A further way of densification is an **increase of the extent of sectorization**. That is, the number of sectors which are served by one and the same antenna site and consequently the number of co-located BSs, each serving one sector, is increased. In contrast to heterogeneous cellular networks, different BS types are not required, which simplifies maintenance. Furthermore, already operated antenna sites may be reused, which eliminates the challenging necessity to open new antenna sites. However, depending on the deployment and transmission concept, the need for adapting the radiation characteristics of the BS antennas may arise. Similarly to the MIMO case, changes of the number of antennas per site and changes of the antenna radiation characteristics may require modifications in rental agreements and may as well aggravate the social acceptance of antenna sites. Like the heterogeneity approach, the sectorization approach allows to increase the network capacity [RRMF10, RF11].

### Coordinated Multi-Point (CoMP)

A completely different but promising approach to increase the network capacity and the homogeneity of the user performance is to introduce coordination or even cooperation of multiple BSs which is commonly referred to as coordinated multi-point (CoMP) [MF11].

Based on the known rationale that known interference can be exploited [Cos83], CoMP allows multiple BSs to jointly transmit or jointly receive data. Among others, Vishwanath et al. and Jindal et al., and Weingarten et al. showed in [VJG03, JVG04, WSS04, WSS06], the theoretic potential of CoMP. In the meantime, also various field trials [HRF11, IDM<sup>+</sup>11] demonstrated the practically achievable increase of the capacity and the improved homogeneity of the user performance. Due to their big potential, CoMP techniques are planned to be used in upcoming cellular networks such as LTE-Advanced [3GP10, 3GP13].

However, the promising CoMP gains come at increased costs. Clearly, to exploit the interference it has to be known. For this purpose, known symbols, so-called pilots, are additionally transmitted and evaluated at the receiver side. In the downlink, the BSs can estimate the channel and interference characteristics based on these pilots and apply the resulting channel state information (CSI) to predistort or precode the transmit symbols. On the opposite side, in the uplink the BSs can apply CSI to detect or decode the distorted received symbols. Obviously, the number of required pilots increases with the number of propagation paths to estimate. Consequently, the necessary pilot overhead increases with the number of coordinated BSs.

To ensure CSI availability at the BSs in the downlink, in a time division duplex system, channel reciprocity can be exploited, i.e., the BSs can reuse their computed channel estimates of the uplink transmission for the downlink transmission. By contrast, in frequency division duplex systems, the mobile stations (MSs) have to compute the channel estimates on their own, which increases their computational effort. Furthermore, the MSs have to feed either the CSI or derived parameters back to the BSs.

To finally enable real-time joint downlink transmission or joint uplink reception, the CSI as well as scheduling and signaling information, and possibly user data have to be available at all cooperating BSs. Thus, a backhaul infrastructure with high data rate and low latency between cooperating BSs is required.

Available backhaul technologies comprise fiber-, copper-, and microwave-based solutions. Very-High-Speed Digital Subscriber Line 2 (VDSL2) is a hybrid solution overcoming the distance between the core network and the street cabinet applying fiber and using copper wire between the street cabinet and the BS. According to [MF11], this technology may be less promising for certain CoMP schemes due to the limited achievable data rate as well as a possibly inappropriate latency on the order of 1 ms. As opposed to this, microwave links with their typical latency of  $100 \mu\text{s}$  [MF11], may comply with CoMP latency requirements. Although the achievable data rate on the order of 1 Gbit/s [MF11] exceeds the achievable backhaul rate of VDSL2, it may still be critical. Further shortcomings of the widely deployed microwave links are their limited reach, high maintenance costs and the restriction to line-of-sight (LOS) conditions. Moreover, the actual data rate depends on the weather conditions. As opposed to this, fiber-based backhaul solutions such as Ethernet [IEE10] and 10-Gigabit-capable passive optical networks (XG-PON) [ITU10] offer a very good error performance and remarkably higher data rates. While XG-PON already enables up to 10 Gbit/s [ITU10] with a latency of about  $100 \mu\text{s}$  [MF11], Ethernet even allows up to

100 Gbit/s [IEE10] with a latency of only a few microseconds [MF11]. However, compared to microwave backhaul links, fiber backhaul links cause high installation costs.

The overall required signal processing complexity depends on the type of coordination or cooperation. While the effort of a limited coordination remains moderate, the signal processing complexity of a full BS cooperation with joint transmission or reception increases tremendously with the number of cooperating BSs, the so-called cluster size.

Of course, all cooperating BSs may be driven by different clocks. Furthermore, different delays may arise from different distances to the user equipments (UEs). Consequently, CoMP requires additional advanced synchronization techniques.

### **Intra-Site CoMP (ISC)**

The original concept of CoMP allows an inter-site cooperation, i.e., a cooperation of BSs from distant antenna sites. Thus, it enables performance improvements especially at the sector edges between cooperating sites. In contrast to that, intra-site coordinated multi-point (ISC), i.e., the cooperation of BSs which are co-located at a joint antenna site, allows to increase the throughput especially at the sector edges between the sectors of co-located BSs.

The restriction to intra-site coordinated multi-point (ISC) still allows to realize remarkable cooperation gains. However, as opposed to full CoMP, it circumvents the need for providing a high-rate backhaul infrastructure between distant antenna sites. As all cooperating entities are co-located, neither the required backhaul data rate nor the backhaul delay are posing unsolvable challenges. Furthermore, as all cooperating BSs might be driven by the same clock, they could be perfectly synchronized. Thus, ISC is a concept to first pick the “low-hanging fruits” of full CoMP.

## **1.3 Focus of this Thesis**

Operators of sectorized cellular networks aim at a significantly increased capacity as well as an improved homogeneity of the achievable user rates. Considering the relative price drop for mobile cellular services, the costs shall thereby not increase.

As discussed in the previous section, multiple approaches to face these challenges exist. Fortunately, these approaches may complement each other. Within this thesis we focus on the investigation of the opportunities and trade-offs in the design of homogeneous sectorized cellular networks in the light of intra-site coordinated multi-point transmission (highlighted in Fig. 1.1).

Thereby, we focus on the question whether the possible consideration of ISC leads to new design guidelines for sectorized cellular networks. So far, these networks were designed such that the coverage areas of adjacent sectors overlapped just enough to allow seamless handoffs. Conventionally, antenna sites were either equipped with a single base station with omnidirectional antennas or they were equipped with three BSs with directional antennas. If operators aimed at a higher network capacity, they increased the



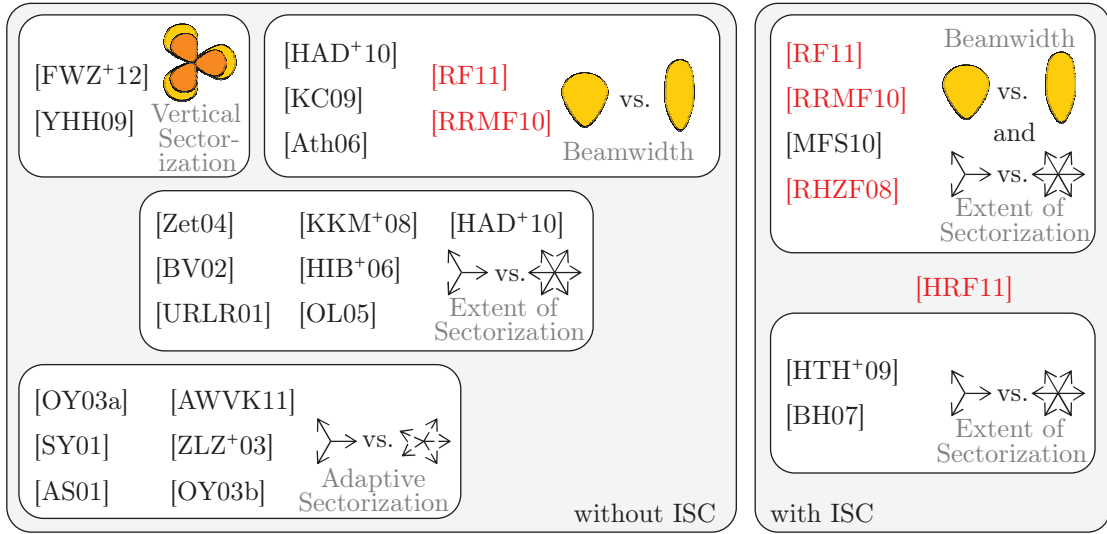


Figure 1.2: Overview of related work on sectorized cellular networks with and without intra-site coordinated multi-point (ISC).

extent of sectorization. That is, they doubled the number BSs per antenna site and the corresponding number of sectors and halved the beamwidth of the respective antennas.

Considering intra-site cooperation, the underlying sectorized cellular system will be deployed in a different way. While conventionally, co-located BSs just share a joint antenna site without cooperation in signal processing, the investigated ISC involves an actual cooperation of the co-located BSs. Obviously, the conventional design paradigm of sector separation by a proper choice of antenna characteristics, will not tap the full potential of the ISC. Hence, this thesis revisits the degrees-of-freedom in the respective sectorized cellular system design: the extent of sectorization, the beamwidth of the applied antennas and the degree of cooperation.

Thereto, a generalized approach to model sectorized cellular systems as well as a suitable BS antenna model are proposed. Consequently, these are applied to revisit the sectorized cellular system design aiming at an increased network capacity as well as an improved homogeneity of the achievable user rates.

## 1.4 Related Work

This section reviews related work on sectorized cellular networks with and without ISC. Fig. 1.2 provides an overview on the contributions grouped according to their main focus. The work of the author of this thesis is marked in red.

### Sectorized Cellular Networks Without ISC

Regarding sectorization in conventional non-cooperative cellular networks, a lot of research results addressing in particular 3- and 6-fold sectorization exist. In 2002, Babich and



Vatta [BV02] showed that 3-fold sectorization increases the area spectral efficiency as compared to non-sectorized cellular networks with omnidirectional antennas. Likewise, Uc-Ríos and Lara-Rodríguez [URLR01] studied the capacity gain when changing from sites with omnidirectional antennas to 3-fold sectorized sites. As opposed to Babich and Vatta, they considered an antenna pattern incorporating a main lobe and a side lobe level leading to inter-sector interference.

Among others, Zetterberg [Zet04], Osseiran and Logothetis [OL05], Hagerman et al. [HIB+06], and Kumar et al. [KKM+08] investigated the capacity gains achievable by further increasing the extent of sectorization.

Moreover, various work especially on the optimization of the antenna beamwidth exists. Kelif and Coupechoux [KC09] presented a closed-form description of an inter-sector interference factor. Based on a fluid model and simplifying assumptions for the antennas, they derived an optimum antenna beamwidth to support a maximum mobile density in a 3-fold sectorized cellular network. Likewise, Athley [Ath06] investigated the optimal azimuthal half-power beamwidth (HPBW) for 3- and 6-fold sectorization in a wideband code division multiple access (WCDMA) network. Thereby, the objective was to minimize the downlink (DL) transmit power such that each MS meets a target signal-to-interference-and-noise ratio (SINR). As opposed to this, Huang et al. [HAD+10] assumed a fixed transmit power per site, and aimed at increasing the throughput per antenna site. For this purpose, they investigated the impact of the extent of sectorization as well as the impact of the antenna beamwidth. However, like Athley, they neglected the physical relationship between the beamwidth and the resulting antenna gain. While Huang et al. completely neglected the antenna gain, Athley chose it depending on the extent of sectorization. In contrast to this, our work addressing the throughput per antenna site in non-cooperative sectorized cellular networks [RRMF10, RF11], has incorporated the dependency of the antenna gain on the antenna beamwidth.

Further approaches especially aim at scenarios with non-uniform user distribution in code division multiple access (CDMA) systems. Anpalagan et al. [AS01] applied fixed overlapping sectors to increase the flexibility of assigning users to base stations. In contrast to that, Saraydar and Yener [SY01] assumed no inter-sector interference and addressed the so-called adaptive cell sectorization. They showed possible power savings if users are served with adaptive non-uniform sectors which are determined depending on the user locations. In [OY03a, OY03b], Oh and Yener extended this work assuming multiuser detection at the antenna site. They investigated the sector arrangements minimizing the total transmit power while ensuring a required signal-to-interference ratio (SIR). As opposed to these optimum solutions, Zhang et al. [ZLZ+03] proposed a suboptimal solution with lower complexity especially in high-density cases.

In 2011, Awada et al. [AWVK11] proposed an iterative algorithm for offline network optimization which jointly optimized the azimuthal boresight direction and the downtilt in non-cooperative sectorized cellular networks.

Another interesting approach is the deployment of additional antenna elements at the antenna site to enable so-called  $3 \times 2$  vertical sectorization [YHH09, FWZ+12]. Thereby,

the co-located antenna elements of an antenna site are operated such that different beams, which are separated by beamwidth and downtilt, serve inner and outer sectors.

### Sectorized Cellular Networks with ISC

Further work in this area already addresses N-fold sectorization in conjunction with ISC. In our work [RHZF08], we have compared uncoded bit error rates of three different site setups with six antennas per site and cooperative preprocessing at the antenna site. In [BH07], Boccardi and Huang compared the throughput of sites with 12 antennas with different configurations applying conventional non-coordinated transmission as well as intra-site and inter-site coordination. Likewise, Huang et al. [HTH<sup>+</sup>09] evaluated such a scenario. Both showed that coordinated transmission may outperform conventional non-coordinated transmission also in the case of higher-order sectorization. Thereby, they studied the impact of different sector configurations with antenna properties chosen such that the coverage areas did not overlap adjacent sectors. Furthermore, they neglected the impact of the antenna gain.

As opposed to this, our work in [RRMF10, RF11] incorporated the impact of the antenna gain and addressed scenarios with overlapping coverage areas. In 2011, our DL cellular field trials [HRF11] confirmed the particular benefit sector edge users have from cooperation.

Müller et al. [MFS10] evaluated the performance of intra-site joint detection and joint link adaption in a 3-fold sectorized network deployment. As they incorporated a 2-dimensional antenna model and considered the beamwidth-dependent antenna gain, their work represents a counterpart to our work addressing the DL [RF11].

## 1.5 Outline and Overview of Contributions

As discussed in the previous section various work aiming at an increased network capacity in sectorized cellular networks exists. However, there are only a few contributions considering the opportunities of cooperative transmission or reception. The aim of this thesis is to broaden the understanding of sectorized cellular networks when considering non-cooperative as well as cooperative transmission. The remainder of this thesis is structured as follows:

**Chapter 2** addresses transmission in sectorized cellular networks. After introducing general elements forming sectorized cellular networks, Section 2.1 discusses the conventionally applied concept of sectorized cellular systems and describes the generalized concept developed within this thesis. In contrast to the conventional concept, our generalized concept allows investigating arbitrary extents of sectorization and enables to scrutinize the effects of sectors with overlapping coverage areas. The subsequent Section 2.2 characterizes antennas applied in sectorized cellular networks and describes the antenna models developed and applied within this thesis. As opposed to the antenna models applied in state-of-the-art system level analyzes addressing sectorized cellular systems, our derived BS antenna

model comprises an antenna pattern representing a main lobe, a level of side and back lobes and considers the derived antenna gain. This is needed for a profound analysis of the impact of different beamwidths of BS antennas. Finally, Section 2.3 discusses the applied transmission model and introduces the considered non-cooperative as well as cooperative transmission concepts. The concepts have been chosen such that the potential of ISC schemes in sectorized cellular networks can be evaluated.

**Chapter 3** presents a system level analysis of non-cooperative as well as cooperative transmission in sectorized cellular networks. After introducing the methodology of the system level analysis, the sectorized cellular network layout is discussed. Various contributions investigating the extent of sectorization exist. However, almost all address non-cooperative systems (cf. Section 1.4). Thus, Section 3.3 complements the common understanding by applying the generalized approach to model sectorized cellular networks as well as the enhanced BS antenna model to consider non-cooperative as well as cooperative transmission concepts. The results suggest that in the case of no intra-site cooperation, the performance increases with higher extents of sectorization. However, the achievable gains decrease with increasing extents of sectorization. As opposed to this, cooperative concepts still benefit from increasing extents of sectorization. Furthermore, in Section 3.4 this thesis adds to the state-of-the-art by investigating the impact of possible antenna beamwidths on two major objectives of mobile network operators, i.e., the network throughput and the QoE relevant user throughput and fairness among users. Subsequently, Section 3.5 and Section 3.6 discuss the sensitivity to the system load and the impact of the degree of cooperation. After addressing the extension of our results to sectors with multi-antenna base stations in Section 3.7, Section 3.8 concludes this chapter with a summary of the main results.

**Chapter 4** discusses practical implications of the key findings of this thesis. Thus, we address implications for operating sectorized cellular networks, and debate on possible changes of the future equipment as well as discuss further implications. One of the key implications is that intra-site cooperation renders  $70^\circ$  antennas, i.e. antennas typically applied at 3-sector sites, also attractive for the application at antenna sites with higher extents of sectorization. Furthermore, the introduction of ISC allows flexible scaling of the activity of BSs depending on the instantaneous traffic demand.

Finally, **Chapter 5** summarizes the main contributions and key findings and proposes possible future research directions.

## 1.6 Notation

The following notation is applied throughout this thesis:

- ▷ Bold lowercase letters  $\mathbf{a}$  and bold uppercase letters  $\mathbf{A}$  denote vectors and matrices, respectively.
- ▷ Uppercase calligraphic letters  $\mathcal{A}$  denote special sets.

- ▷ The sets of real and complex numbers are denoted by  $\mathbb{R}$  and  $\mathbb{C}$ .
- ▷  $\mathbf{I}_N$  denotes an  $N \times N$  identity matrix.
- ▷ The notation  $\mathbf{H} \in \mathbb{C}^{[K \times N]}$  is applied to denote the complex  $K \times N$  matrix  $\mathbf{H}$ .
- ▷ The entry in the  $k$ th row and  $n$ th column of a matrix  $\mathbf{H}$  is denoted by  $h_{k,n}$  or  $[\mathbf{H}]_{k,n}$ , as appropriate. Moreover, the notation  $[\mathbf{H}]_{k,:}$  denotes the submatrix of  $\mathbf{H}$  comprising all entries of the  $k$ th row of  $\mathbf{H}$ .
- ▷ The  $k$ th element of a vector  $\mathbf{a}$  is denoted by  $a_k$ .
- ▷  $(\cdot)^T$ ,  $(\cdot)^*$  and  $(\cdot)^\dagger$  denote the transpose, the complex conjugate and the complex conjugate transpose, respectively.
- ▷ The operator  $|\cdot|$  denotes the absolute value and the cardinality when applied to numbers and sets, respectively.
- ▷ The operator  $\|\cdot\|$  denotes the Euclidean vector norm.
- ▷  $\text{tr}\{\cdot\}$  and  $\text{E}\{\cdot\}$  denote the trace operation and the expectation, respectively.
- ▷ The operator  $\det\{\cdot\}$  is applied to denote the determinant.
- ▷  $\Phi_{\mathbf{x}\mathbf{x}} = \text{E}\{\mathbf{x}\mathbf{x}^\dagger\}$  denotes the covariance matrix of vector  $\mathbf{x}$ .