

Marco Krondorf  
Analytical Methods for Multicarrier  
Performance Evaluation



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**Marco Krondorf**

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**Technische Universität Dresden**

# **Analytical Methods for Multicarrier Performance Evaluation**

**Marco Krondorf**

der

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Prof. Dr.-Ing. Eduard Jorswieck

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## Abstract

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This thesis extends state of the art OFDM link and system level performance evaluation methods, motivated by the practical relevance of the OFDM concept. The enormous system parameter space of practical OFDM implementations necessitates the development of evaluation methods which are mainly based on analytical or numerical calculations instead of time consuming Monte Carlo simulations. The investigated methods include the simultaneous effect of various transceiver impairments - so called Dirty RF effects, where the presented framework goes beyond the state of the art in OFDM performance analysis where such impairments are typically treated separately. Special focus is set on effects originating from user mobility and on the impact of the system's carrier frequency - two parameters which are shown in this thesis to greatly affect the global system performance.

The thesis introduces OFDM principles and explains the essential building blocks and baseband algorithms of an OFDM transceiver chain. In this respect, an equivalent OFDM baseband signal model is developed which accounts for the simultaneous effects of the mentioned impairments. Subsequently, OFDM performance metrics are detailed and the mobile channel model is presented where it is shown that the entire OFDM Tx-Rx chain can be described by means of a conditional probability density function (PDF). This conditional PDF can finally be used for performance metric computations, leading to bit error rate and spectral efficiency evaluation results.

Moreover, the thesis explains how to derive the conditional PDF for a variety of OFDM link setups. As an interesting result, the conditional PDF for various multi-antenna concepts has a common structure where the diversity degree directly affects the PDF. At the same time, the PDF is shown to be highly dependent on the channel estimation scheme and the OFDM pilot structure. In this respect, the performance loss due to outdated channel state information (CSI) at the receiver side is analytically evaluated and exhibits a strong impact on the overall system performance. Typically outdated CSI originates from imperfect CSI tracking in time-varying channels. The tracking accuracy is typically quantified as cross-correlation coefficient of the instantaneous channel and the CSI used for OFDM signal equalization. Here, a cross-correlation of one means perfect tracking. The thesis shows that even slightest decorrelations

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away from one introduce significant performance losses which tend to overwhelm those losses originating from Dirty RF.

As a major outcome, the results clearly indicate a distinction between carrier frequency dependent and non-dependent impairments. Here, phase noise and outdated CSI are identified as frequency dependent which offers interesting study items when extending the link level performance evaluation framework toward system level studies. The latter is done in this thesis for a LTE-based cellular infrastructure where the carrier frequency dependent spectral efficiency is calculated for different user mobility profiles and system specifications. The numerical results indicate the notion of a best suited carrier frequency in terms of spectral efficiency maximization for a given user mobility. At the same time, the results hint the necessity of flexible spectrum allocation schemes as enablers of further growth in the wireless communications sector.



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## Zusammenfassung

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Die vorliegende Arbeit behandelt die Thematik der numerischen Leistungsfähigkeitsbewertung von OFDM Systemen, wobei Methoden nach dem Stand der Technik erweitert bzw. neue Berechnungsverfahren entwickelt wurden. Bei dem vorgestellten Framework geht es im Kern darum, OFDM-Verbindungen hinsichtlich ihrer Übertragungskapazität zu untersuchen, wenn eine Reihe praktischer Störeinflüsse das System beeinflussen. Die entwickelten Berechnungsverfahren sind dabei vielseitig anwendbar und berücksichtigen eine Reihe gleichzeitig wirksamer Störeinflüsse wie:

- restlicher Trägerfrequenzversatz nach imperfekter Synchronisation
- Kanalschätzfehler unter Berücksichtigung verschiedener Pilotsignalanordnungen und Kanalschätzalgorithmen
- I/Q Imbalance, Phasenrauschen, sendeseitige PA-Nichtlinearität, begrenzte ADC Auflösung
- Nutzermobilität und ungenaues empfangsseitiges Nachführen der Kanalinformation

Der Hauptteil der Arbeit beschreibt die mathematischen Grundlagen des Frameworks für SISO, MISO & SIMO OFDM Verbindungen und untersucht das Systemverhalten im Falle verschiedener Störeinflüsse. Besonders ist dabei der Effekt nicht perfekt nachgeführter Kanalinformation herauszuheben, welcher bei hinreichend hoher Nutzermobilität bedeutende Performanceeinbrüche mit sich bringen kann. Diese Aussage erscheinen auf den ersten Blick als schon oft untersucht bzw. trivial, allerdings steckt hier die Neuigkeitswert in der stochastischen Signalbeschreibung welche hochgenaue Performanceanalysen zulässt.

Die numerischen Verfahren zur Berechnung relevanter OFDM Leistungskenngrößen basieren dabei auf geeignet formulierten OFDM-Basisbandsignalmodellen und deren stochastischer Modellierung mittels einer Wahrscheinlichkeitsdichtefunktion. Im Rahmen der Arbeit wird gezeigt, dass sich eine Vielzahl von OFDM-Mehrantennenkonfigurationen durch die gleiche parametrisierte Dichtefunktion beschreiben lassen, welche direkt vom Diversitätsgrad des Systems abhängt.

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Das so entstandene Link-Level Framework kann, sofern entsprechend konfiguriert, nun in System-Level Simulationen eingesetzt werden, um das makroskopische Verhalten OFDM basierter Funkzugangsnetze zu untersuchen. Dabei liefert das Framework die Spektrale Effizienz bzw. die Spektrale Effizienz pro versorgter Fläche als relevante Leistungskenngrößen. Dementsprechend werden die entwickelten Berechnungsverfahren im letzten Teil der Arbeit auf ein zellulares LTE-Mobilfunksystem angewendet. In den gezeigten Untersuchungen wird dabei die Trägerfrequenz- sowie die Nutzermobilitätsproblematik in den Vordergrund gestellt. Die numerischen Berechnungen zeigen eine deutliche Abhängigkeit der spektralen Effizienz von zwei Systemparametern: (1) der Trägerfrequenz sowie (2) der Mobilität der Mobilfunknutzer. Weiterhin zeigt die Arbeit, wie sich verschiedene Techniken der Kapazitätssteigerungen auf das zellulare Funkzugangsnetz auswirken. Herausgehoben sie hierbei der geteilte Frequenzwiederholfaktor (engl. fractional frequency reuse - FFR) welcher durch trägerfrequenzabhängige Parametrisierung die spektrale Effizienz deutlich beeinflusst.

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## Nomenclature

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### Notation

$n$	lowercase letters indicate scalars, events of a random variable and time domain signals
$X$	uppercase letters denote random variables and frequency domain signals
$\mathbf{M}$	boldface characters denote matrices and vectors
$\mathbf{M}_{[r \times c]}$	indicates the dimension of matrix $\mathbf{M}$ having $r$ rows and $c$ columns
$[\mathbf{M}]_{i,j}$	denotes the element in the $i$ -th row and $j$ -th column of matrix $\mathbf{M}$

### Symbols

$a_l, b_l$	marginal PDF parameter on subcarrier $l$
$A_1$	reuse 1 cell area
$A_{\text{cell}}$	total cell area
$A$	clipping level for the soft-limiter nonlinearity, $A$ is also used to represent amplitudes of signals
$\mathbf{B}$	frequency domain channel estimation/interpolation matrix
$c_{\text{vco}}$	oscillator constant
$C$	coverage of a radio cell averaged over log-normal shadowing
$d$	time domain distortion noise from the Bussgang theorem
$D_l$	frequency domain distortion noise on subcarrier $l$
$D$	diversity, reuse distance if used in the context of cellular deployments
$D_t, D_f$	pilot carrier distance in time and frequency direction
$\mathbf{E}$	frequency domain carrier cross talk matrix
$E\{\cdot\}$	stochastic expectation operator
$f_c$	carrier frequency
$f_{D,\text{max}}, f_{D,\text{norm}}$	maximum Doppler frequency, normalized Doppler frequency
$h_\tau$	channel impulse response tap at position $\tau$

$G$	antenna gain in dB
$H_l$	channel transfer function coefficient on subcarrier $l$
$\mathbf{H}$	frequency domain channel vector
$j$	imaginary unit
$k$	subcarrier index
$K$	I/Q Imbalance leakage constant
$l$	subcarrier index
$L$	channel impulse response length
$m$	subcarrier index for the I/Q Imbalance leakage signal
$M$	order of the QAM scheme, link budget margins if used with indexing
$\mathbf{M}$	auxiliary matrix for channel estimation error modeling
$n$	sample index for time domain signals
$N, N_C, N_{CP}$	FFT size, number of used subcarriers, number of cyclic prefix samples
$N_S$	number of sectors per cell
$p, q$	random variables used to describe the stochastic data symbol
$P_{\text{loss}}$	path loss
$P_{\text{Tx}}$	transmit power
$r$	random variable to describe distances and magnitudes
$R$	cell radius
$t$	continuous time
$T$	throughput used with various indices, spectral efficiency
$T_{\text{area}}$	area spectral efficiency
$T_{\text{Sym}}, T_U, T_{CP}$	duration of the entire OFDM symbol, duration of the useful OFDM symbol part, cyclic prefix duration
$v$	user velocity, VCO response when function of time $t$
$w, W$	time and frequency domain additive Gaussian noise
$x, X$	time and frequency domain transmit signal
$y, Y$	time and frequency domain receive signal
$Z_l$	equalized frequency domain signal on subcarrier $l$
$\alpha, \beta, \varphi, \Phi$	used to represent angles and phases
$\beta$	Tx power normalization factor under nonlinear transmit signal distortions
$\gamma$	SNR and SINR per carrier averaged over fast fading if not otherwise stated
$\kappa$	scaling factor from the Bussgang theorem
$\mu$	FFR ratio

$\nu$	CFO normalized to the subcarrier spacing
$\rho$	constant to represent the IBO, system load if used in the context of cellular deployments
$\sigma^2$	used to represent variances
$\tau$	used to represent time lag parameters

### Functions

$C(\gamma)$	Shannon capacity given SNR $\gamma$
$\exp\{\cdot\}$	exponential function $e^{(\cdot)}$
$g(\cdot)$	nonlinear amplitude distortion characteristic
$p_R(r)$	probability density function of RV $R$ at realization $r$
$\Pr(\cdot)$	probability of a given event
$Q(\cdot)$	Gaussian Q function
$R(\gamma)$	mutual information at SNR $\gamma$

### Abbreviations

3GPP	third generation partnership project
ADC	analog-to-digital conversion
AGC	automatic gain control
BS	base station
BER	bit error rate
BP	band pass
CFO	carrier frequency offset
CIR	channel impulse response
CP	cyclic prefix
CDI	channel distribution information
CSI	channel state information
CSMA/CA	carrier sense multiple access / collision avoidance
DAC	digital-to-analog conversion
DC	direct current
(I)DFT	(inverse) discrete Fourier transform
DL, UL	downlink, uplink
EIRP	equivalent isotropic radiated power
FEC	forward error control (channel coding)

FFR	fractional frequency reuse
(I)FFT	(inverse) fast Fourier transform
IBO	input power backoff
ICI	inter carrier interference
IF	intermediate frequency
LOS , NLOS	line of sight, non line of sight
LP	low pass
LTE	the 3GPP Long Term Evolution radio standard
LTS	long training symbol
MAC	medium access control
MISO	multiple input single output
MRC	maximum ratio combining
MS	mobile station
OFDM	orthogonal frequency division multiplexing
PA	power amplifier
PAPR	peak-to-average power ratio
PDF	probability density function
PDP	power delay profile
PHY	physical layer
PLL	phase locked loop
PN	phase noise
QAM	quadrature amplitude modulation
QoS	quality of service
RV	random variable
SER	symbol error rate
SISO, SIMO	single input single output, single input multiple output
SNR, SINR	signal-to-noise ration, signal-to-interference and noise ratio
STC	space time coding
STS	short training symbol
VCO	voltage controlled oscillator
WiMAX	wireless interoperability for microwave access
WLAN	wireless local area network





# CHAPTER 1

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## Introduction

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### 1.1. Mobile Wireless Communications

The evolution of modern industrialized countries to knowledge and information based societies goes hand in hand with far reaching changes in the telecommunications sector. Due to the vast rollout of internet technologies in the early 90s, the instantaneous broadband data access becomes an enabler for economic success of globally operating companies. At the same time, the computational power of IT components evolved which finally led to the *Digitization of Communications* as referred to by R. Olexa in [Ole05]. Pure voice or data services were substituted by multi media applications where video and audio data is provided to a large group of subscribers. Throughout the past decade, the internet gets equivalent to the classical telephone services in terms of distribution and economic relevance. This fact is especially visible in the evolution of circuit switched IT infrastructures, used mainly for voice transmission, to packet switched networks.

In parallel to the internet evolution of the 90s, 2nd generation digital cellular networks were rolled out in the developed industrial countries. Here, the well known European GSM standard is the worldwide most successful 2G cellular system and even today serves as a reference in terms of subscriber count and return of invest. The offensive marketing of mobile operators and content providers turned the term *Mobility* into a synonym for availability, independence, individuality and last but not least for economic and private success.

The intense penetration of mobile applications and services goes in line with ever growing user expectations in terms of availability and bandwidth. A logical consequence is the ongoing development of wireless standards and access technologies. A successful growth in the wireless sector is indispensably linked with rich expertise in the fields of RF engineering, PHY design, radio wave propagation and cellular interference management.

In order to provide reliable high-bandwidth services, today's and especially future wireless

systems employ the multi carrier concept OFDM - *Orthogonal Frequency Division Multiplexing*. OFDM is known to enable high spectral efficiency and low complexity equalization. There is a number of standards even today which implement OFDM: DVB-T (TV broadcast), DAB and DRM (audio broadcast), IEEE 802.11a/n (WLAN) and IEEE 802.22 (WRAN). The upcoming 4G mobile communications systems IEEE 802.16e (mobile WiMAX) and 3GPP LTE (Long Term Evolution) use OFDM and OFDM derivatives like OFDMA as well.

## 1.2. Thesis Outline and Contribution

This thesis extends available OFDM link and system level performance evaluation methods, motivated by the practical relevance of the OFDM concept. The enormous system parameter space of practical OFDM implementations necessitates the development of evaluation methods which are mainly based on analytical or numerical calculations instead of time consuming Monte Carlo simulations. The investigated methods include the *simultaneous* effect of various transceiver imperfections - so called *Dirty RF* [DRF07] effects. Hence, it goes far beyond the state of the art in OFDM performance analysis and prediction where such impairments are typically treated separately. Special focus is set on the effects due to user mobility and on the impact of the system's carrier frequency - two parameters which are shown in this thesis to greatly affect the global system performance.

The outline of this thesis is as follows. In Chapter 2 we introduce OFDM principles and explain the essential building blocks and baseband algorithms of an OFDM transceiver chain. OFDM performance metrics are detailed and the mobile channel model is presented.

Chapter 3 treats the development of the mentioned analytical and numerical OFDM performance analysis methods. The scope of this chapter is to derive conditional probability density functions (PDF) for the received signal which are then used for performance metric computations.

In chapter 4 we extend the PDF derivations of Chapter 3 toward scattered pilot grids . Furthermore, we introduce the interesting study items of carrier frequency dependent OFDM throughput as well as optimized OFDM pilot patterns to address user mobility effects. The final Chapter 5 applies the link level performance analysis framework on cellular 3GPP LTE deployments in order to derive both spectral efficiency and area spectral efficiency. The numerical results indicate the notion of a *best suited carrier frequency* in terms of spectral efficiency maximization. At the same time, the results hint the necessity of flexible spectrum allocation schemes as enablers of further growth in the wireless communications sector.