

Patrick Marsch

Coordinated Multi-Point under a Constrained Backhaul and  
Imperfect Channel Knowledge



Beiträge aus der Informationstechnik

Mobile Nachrichtenübertragung

Nr. 49

**Patrick Marsch**

**Coordinated Multi-Point under a  
Constrained Backhaul and  
Imperfect Channel Knowledge**

 VOGT

Dresden 2010

Bibliografische Information der Deutschen Bibliothek  
Die Deutsche Bibliothek verzeichnet diese Publikation in der Deutschen  
Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über  
<http://dnb.ddb.de> abrufbar.

Bibliographic Information published by Die Deutsche Bibliothek  
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;  
detailed bibliographic data is available in the internet at <http://dnb.ddb.de>.

Zugl.: Dresden, Techn. Univ., Diss., 2010

Die vorliegende Arbeit stimmt mit dem Original der Dissertation  
„Coordinated Multi-Point under a Constrained Backhaul and  
Imperfect Channel Knowledge“ von Patrick Marsch überein.

© Jörg Vogt Verlag 2010  
Alle Rechte vorbehalten. All rights reserved.

Gesetzt vom Autor  
Printed in Germany

ISBN 978-3-938860-35-9

Jörg Vogt Verlag  
Niederwaldstr. 36  
01277 Dresden  
Germany

Phone: +49-(0)351-31403921  
Telefax: +49-(0)351-31403918  
e-mail: [info@vogtverlag.de](mailto:info@vogtverlag.de)  
Internet : [www.vogtverlag.de](http://www.vogtverlag.de)

Technische Universität Dresden

**Coordinated Multi-Point under a Constrained Backhaul and  
Imperfect Channel Knowledge**

Patrick Marsch

der Fakultät Elektrotechnik und Informationstechnik  
der Technischen Universität Dresden

zur Erlangung des akademischen Grades eines

**Doktoringenieurs**

(Dr.-Ing.)

genehmigte Dissertation

Vorsitzender: Prof. Dr.-Ing. habil. Rüdiger Hoffmann

Gutachter: Prof. Dr.-Ing. Gerhard Fettweis

Prof. Dr. rer. nat. Rudolf Mathar

Tag der Einreichung: 7. Dezember 2009

Tag der Verteidigung: 1. März 2010

Dresden, 20. Mai 2010

# Abstract

Mobile communication has become an essential part of today's information society. Especially the demand for ubiquitous mobile Internet access has significantly increased in the past years, creating a severe challenge for mobile operators to respond to the demand for mobile data rates, while at the same time strongly reducing cost per bit. This challenge can only be successfully addressed if the spectral efficiency and fairness of mobile communications are continuously increased. In today's cellular systems, both aspects are more and more limited through the interference between cells, especially in dense urban deployments.

From theory it is known that this inter-cell-interference can be canceled or even exploited if base stations cooperatively process signals connected to multiple terminals, a concept commonly referred to as *Coordinated Multi-Point* (CoMP). As these schemes promise significant improvements of spectral efficiency and a more homogeneous throughput distribution, they are seen as a key technology of future mobile systems. Beside many implementation challenges, such as the synchronization of the cooperating entities and an accurate estimation of the involved wireless links, a main issue connected to CoMP is the additional infrastructure required for data exchange between cooperating base stations, usually referred to as *backhaul*.

This work provides an information theoretical analysis of the trade-off between capacity gains and required backhaul achievable with various CoMP concepts, also taking into consideration the major impact of imperfect channel knowledge at base station and terminal side. A key finding is that the relative benefit of CoMP in fact increases in certain scenarios under less accurate channel knowledge. Also, major throughput gains are already possible through flexible user assignment and decoding concepts, without requiring any backhaul. For the cellular uplink, two cooperation strategies are identified that should ideally be used adaptively, depending on the current interference situation. One can be used for a very backhaul-efficient, low complexity, decentralized cancelation of weak interference, where the base stations exchange decoded data. In the other, centralized, scheme, the base stations exchange received signals, providing larger gains under stronger interference conditions, but requiring more backhaul. For the downlink, a flexible scheme of moderate complexity is identified that provides a good throughput/backhaul trade-off for most channel conditions. Iterative cooperation concepts are shown to be of minor value, despite several publications in this field.

Beside the analysis of small CoMP scenarios, the work also provides a concept for the backhaul-efficient usage of CoMP in large cellular systems. This concept exploits the fact that co-located base stations can cooperate without requiring backhaul, while smart clustering and resource partitioning concepts can provide further gain at minimal backhaul. This yields a system with strong fairness and capacity gains over a conventional system, while requiring an additional backhaul infrastructure with a capacity less than twice the system capacity.

# Zusammenfassung

Die mobile Kommunikation hat einen enormen Stellenwert in der heutigen Gesellschaft eingenommen. Insbesondere die steigende Nachfrage nach allgegenwärtigem, mobilem Internetzugang stellt Netzbetreiber zunehmend vor die Herausforderung, flächendeckend höhere Datenraten anzubieten, bei gleichzeitig verringerten Kosten pro Bit. Hierzu muß die spektrale Effizienz und Fairness von Mobilfunksystemen konsequent verbessert werden, die in heutigen Systemen primär durch die Interferenz zwischen benachbarten Zellen beschränkt ist.

Aus der Theorie ist bekannt, dass Inter-Zellen-Interferenz reduziert oder sogar ausgenutzt werden kann, wenn Basisstationen kooperativ die Signale mehrerer Endgeräte verarbeiten. Diese so genannten *Coordinated Multi-Point* (CoMP) Verfahren versprechen erhebliche Steigerungen und eine homogenere Verteilung von Datenraten und gelten als Schlüsseltechnologien des Mobilfunks der Zukunft. Neben diversen Herausforderungen bei der Implementierung, z.B. der zellübergreifenden Synchronisation und Kanalschätzung, besteht ein Hauptproblem bei CoMP jedoch darin, dass eine zusätzliche Kommunikationsinfrastruktur zwischen kooperierenden Basisstationen benötigt wird - so genannter *Backhaul*.

Die vorliegende Arbeit führt eine informationstheoretische Analyse des Verhältnisses aus Datenrate und benötigtem Backhaul von verschiedenen Kooperationsstrategien durch, wobei auch der Einfluss fehlerhafter Kanalkennntnis berücksichtigt wird. Eine wesentliche Beobachtung ist, dass der relative Gewinn durch CoMP in bestimmten Szenarien zunimmt, je schlechter die Kanalkennntnis ist. Ferner können innovative Nutzerzuordnungs- und Dekodierkonzepte Kapazitätssteigerungen erzielen, ohne dass Backhaul benötigt wird. Für die zellulare Aufwärtsstrecke werden zwei Kooperationsverfahren identifiziert, zwischen denen ein System idealerweise je nach Interferenzsituation umschaltet. Das erste, dezentralisierte Verfahren erlaubt eine Backhaul-effiziente Reduktion von schwacher Interferenz bei geringer Komplexität. Hierbei werden zwischen den Basisstationen dekodierte Nutzdaten ausgetauscht. Ein zweites, zentralisiertes Verfahren, basierend auf dem Austausch quantisierter Empfangssignale, ist vorteilhaft in Szenarien starker Interferenz, benötigt jedoch mehr Backhaul. In der Abwärtsstrecke wird ein flexibles Verfahren mittlerer Komplexität vorgestellt, das ein gutes Verhältnis aus Datenraten und benötigtem Backhaul für eine Vielzahl von Kanälen ermöglicht. Auch iterative Kooperationsverfahren werden untersucht, erweisen sich jedoch als wenig attraktiv.

Neben der Betrachtung kleiner CoMP Szenarien stellt die Arbeit ein Gesamtkonzept für Backhaul-effizientes CoMP in großen zellularen Systemen vor. Dieses nutzt die Tatsache aus, dass am gleichen Ort befindliche Basisstationen ohne Backhaulbedarf kooperieren können, und verwendet Gruppierungs- und Ressourcenpartitionierungskonzepte, um weitere Kapazitätsgewinne bei geringem Backhaulbedarf zu erzielen. Zudem sind deutliche Fairnessverbesserungen gegenüber herkömmlichen Mobilfunksystemen zu verzeichnen, obwohl auf dem Backhaul lediglich ein der doppelten Systemkapazität entsprechender Datenaustausch erforderlich ist.



# Acknowledgement

This thesis is the result of my work at the Vodafone Chair Mobile Communications Systems at Technische Universität Dresden. I would particularly like to thank Prof. Gerhard Fettweis, who not only convinced me to do a Ph.D. in his team and hence made the work possible, but who also gave me a large number of opportunities and interesting challenges in the past four years that enabled me to improve both academic and personal skills. For example, he gave me the chance to be strongly involved in one of the most renowned research projects connected to next generation mobile communications systems, namely the project EASY-C, partially funded by the German Federal Ministry of Education and Research (BMBF). It was highly interesting and challenging to design a CoMP experimentation platform at a point in time when 3GPP was just only starting the discussion on this topic, and I am of course happy to see that recent field trial results do in fact support many of the statements made in this thesis. I would like to thank Steffen Watzek for the great collaboration, and all EASY-C team members for their continuous enthusiasm, stamina and humor, in particular during the last-minute preparation of several live demonstrations.

Since April 2009, I also had the chance to setup a new research group within the Vodafone Chair, observing system-level aspects of CoMP and heterogeneous cellular deployments, and venturing into the novel viewpoint of energy-efficient communications. I would like to thank the group members for the many fruitful discussions and the great ideas brought up, and I am looking forward to continuing joint work in the upcoming months.

The thesis wouldn't be what it is without the great support of Peter Rost, Michael Grieger, Fabian Diehm and Albrecht Fehske, who rigorously proof-read the document and/or with whom I had many discussions that helped shape the work. Thanks also to Prof. Rudolf Mathar, who provided the second advisory opinion on the thesis and gave me important hints for the defense presentation. I finally want to thank Shahid Khattak and André Fonseca dos Santos for the enlightening time we had when working or traveling together, and Ernesto Zimmermann, who originally motivated me to join the Vodafone Chair.

Last but surely not least, I am deeply grateful to my family and friends for their continuous encouragement and support, and to my dear Ines, whose patience and understanding even in the longest nights of writing up this work was amazing.



# Contents

<b>Abstract / Zusammenfassung</b>	<b>vii</b>
<b>Acknowledgement</b>	<b>ix</b>
<b>Contents</b>	<b>xi</b>
<b>List of Figures</b>	<b>xiv</b>
<b>List of Tables</b>	<b>xv</b>
<b>Abbreviations</b>	<b>xvii</b>
<b>Symbols</b>	<b>xix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation . . . . .	1
1.2 Contribution of this Work . . . . .	2
1.3 Related Work . . . . .	3
1.4 Structure of this Thesis . . . . .	4
1.5 Notation . . . . .	5
<b>2 Information-Theoretic Basics</b>	<b>7</b>
2.1 General Concepts . . . . .	7
2.2 Uplink . . . . .	8
2.2.1 Transmission Model . . . . .	8
2.2.2 Modeling of Imperfect Channel Knowledge . . . . .	10
2.2.3 Capacity Region Under Infinite BS Cooperation . . . . .	11
2.2.4 Capacity Region without BS Cooperation . . . . .	13
2.2.5 Basic Base Station Cooperation Schemes . . . . .	17
2.3 Downlink . . . . .	29
2.3.1 Transmission Model . . . . .	29
2.3.2 Modeling of Imperfect Channel Knowledge . . . . .	30
2.3.3 Capacity Region Under Infinite BS Cooperation . . . . .	32
2.3.4 Capacity Region without BS Cooperation . . . . .	37
2.3.5 Basic Base Station Cooperation Schemes . . . . .	40
2.4 Performance and Backhaul-Constrained Capacity Regions . . . . .	44
2.5 Summary . . . . .	48

<b>3</b>	<b>Information-Theoretic Analysis</b>	<b>49</b>
3.1	Scenarios, Channels and Metrics Considered . . . . .	49
3.1.1	Uplink . . . . .	50
3.1.2	Downlink . . . . .	52
3.1.3	Channel Matrices . . . . .	53
3.1.4	Sum Rate and Common Rate . . . . .	53
3.2	Cut-Set Bound . . . . .	54
3.3	Uplink Analysis . . . . .	55
3.3.1	Capacity Gains through CoMP in the Uplink . . . . .	55
3.3.2	Performance of Uplink CoMP Schemes for Specific Channels . . . . .	59
3.3.3	Benefit of Source Coding and Superposition Coding . . . . .	67
3.3.4	Benefit of Iterative BS Cooperation Schemes . . . . .	68
3.3.5	Choice of Best Coop. Scheme and Cooperation Direction . . . . .	74
3.3.6	Sensitivity of Schemes to Channel Orthogonality and SNR . . . . .	76
3.3.7	Performance of Uplink CoMP Schemes in Scenarios with $M = K = 3$ . . . . .	78
3.3.8	Summary . . . . .	81
3.4	Downlink Analysis . . . . .	82
3.4.1	Capacity Gains through CoMP in the Downlink . . . . .	82
3.4.2	Performance of Downlink CoMP Schemes for Specific Channels . . . . .	88
3.4.3	Benefit of Superposition Coding in the Downlink . . . . .	93
3.4.4	Performance of Downlink CoMP Schemes for Arbitrary Channels . . . . .	94
3.4.5	Choice of Best Cooperation Scheme . . . . .	95
3.4.6	Sensitivity of Schemes to Channel Orthogonality and SNR . . . . .	97
3.4.7	Performance of Downlink CoMP Schemes in Scenarios with $M = K = 3$ . . . . .	98
3.4.8	Summary . . . . .	100
<b>4</b>	<b>System Level Simulation</b>	<b>101</b>
4.1	Simulation Setup . . . . .	101
4.1.1	Channel Model . . . . .	101
4.1.2	Simulation Flow . . . . .	102
4.2	Clustering and Resource Partitioning . . . . .	105
4.2.1	The Benefit of Resource Partitioning . . . . .	109
4.3	Uplink Simulation Results . . . . .	110
4.4	Downlink Simulation Results . . . . .	113
4.5	Summary . . . . .	114
<b>5</b>	<b>Implications on Practical Systems</b>	<b>117</b>
5.1	General Implications of our Work . . . . .	117
5.1.1	Fundamental Trade-Offs in the Context of CoMP . . . . .	117
5.1.2	Mobile Network Organization . . . . .	117
5.1.3	Backhaul Topology . . . . .	118
5.1.4	On the Value of Iterative BS Cooperation Schemes . . . . .	118
5.2	Practical Considerations . . . . .	120
5.2.1	Practical Implementation of BS Cooperation Schemes . . . . .	120
5.2.2	CSI Exchange, ARQ and Complexity Issues . . . . .	121
5.2.3	Scheduling for CoMP . . . . .	125
5.2.4	Ad-hoc BS Cooperation after Transmission . . . . .	126

5.3	Extension of the Work to other Scenarios . . . . .	127
5.3.1	Intra-cell CoMP in the Uplink . . . . .	127
5.3.2	Terminals with more Receive Antennas in the Downlink . . . . .	127
5.4	Summary . . . . .	128
<b>6</b>	<b>Conclusions</b>	<b>131</b>
6.1	Contribution of this Work . . . . .	131
6.2	Main Conclusions . . . . .	131
6.3	Future Outlook . . . . .	133
<b>A</b>	<b>Literature Overview on CoMP</b>	<b>135</b>
<b>B</b>	<b>Proofs connected to Imperfect Channel Knowledge</b>	<b>137</b>
B.1	Modified Transmission Equation for UL under Imp. CSI . . . . .	137
B.2	Modified Transmission Equation for DL under Imp. CSI . . . . .	139
B.3	Downlink SINRs under Imperfect CSI . . . . .	141
<b>C</b>	<b>Proofs connected to Uplink-Downlink Duality</b>	<b>143</b>
C.1	DL SCO Problem with Per-Antenna Power Constraints . . . . .	143
C.2	Equivalence of UL and DL Capacity Region under Per-Ant. Power Constr. . . . .	147
<b>D</b>	<b>Proofs connected to Capacity Regions</b>	<b>149</b>
D.1	Uplink Capacity Region under Infinite BS cooperation . . . . .	149
D.2	UL Capacity Region with DIS/CIF/DAS BS Cooperation . . . . .	150
D.3	Downlink Capacity Region Calculation . . . . .	157
D.4	Downlink Capacity Region with Common Messages . . . . .	160
<b>E</b>	<b>Motivation of Simulation Parameters</b>	<b>163</b>
<b>F</b>	<b>Proofs connected to Specific BS Cooperation Schemes</b>	<b>169</b>
F.1	Proofs Connected to Iterative DIS (I-DIS) . . . . .	169
F.2	Benefit of Iterative DAS (I-DAS) . . . . .	171
F.3	UL DAS with Source Coding Approaching Cut-set Bound . . . . .	172
F.4	Best Cooperation Direction for Uplink DAS . . . . .	174
F.5	Superiority of DIS over CIF . . . . .	176
	<b>Bibliography</b>	<b>179</b>
	<b>Publication List</b>	<b>197</b>
	<b>Curriculum Vitae</b>	<b>200</b>

# List of Figures

2.1	Model of the uplink transmission considered in this work. . . . .	9
2.2	Model of the uplink transmission assuming infinite BS cooperation. . . . .	12
2.3	Uplink capacity region under no or infinite BS cooperation. . . . .	17
2.4	Admissible rate region of Slepian-Wolf source coding [SW73]. . . . .	18
2.5	Illustration of a combined DIS/CIF/DAS base station cooperation strategy. .	22
2.6	Illustration of the downlink transmission considered in this work. . . . .	30
2.7	Downlink capacity region under no or infinite BS cooperation. . . . .	40
2.8	Uplink performance region for different CoMP schemes. . . . .	45
2.9	Backhaul-constrained uplink capacity region for different CoMP schemes. . .	47
2.10	Backhaul-constrained downlink capacity region for different CoMP schemes. .	47
3.1	Scenarios considered in the uplink and downlink analysis in this chapter. . . .	50
3.2	Uplink transmit power and path gain as a function of UE location. . . . .	51
3.3	Illustration of the cut-set bound. . . . .	54
3.4	CoMP gain in uplink scenarios with $M = K = 2$ of average orthogonality. . .	56
3.5	CoMP gain in uplink scenarios with $M = K = 3$ of average orthogonality. . .	58
3.6	Impact of imperfect CSIR and pathloss exponent on uplink CoMP. . . . .	59
3.7	Non-iterative uplink CoMP schemes for $M = K = 2$ analyzed in this chapter.	62
3.8	Sum rate vs. backhaul for uplink CoMP schemes and specific channels. . . . .	66
3.9	Gain through source coding or superposition coding in uplink CoMP. . . . .	68
3.10	Iterative uplink CoMP schemes for $M = K = 2$ analyzed in this chapter. . . .	72
3.11	Benefit of iterative uplink CoMP schemes for $M = K = 2$ . . . . .	73
3.12	Best choice of BS cooperation scheme in uplink CoMP. . . . .	75
3.13	Impact of channel orthogonality and SNR on uplink CoMP schemes. . . . .	77
3.14	Monte Carlo simulation results for $M = K = 3$ in the uplink. . . . .	80
3.15	CoMP gain in downlink scenarios with $M = K = 2$ of average orthogonality.	84
3.16	CoMP gain in downlink scenarios with $M = K = 3$ of average orthogonality.	86
3.17	Impact of imperfect CSIT and CSIR on downlink CoMP. . . . .	87
3.18	Downlink CoMP schemes for $M = K = 2$ analyzed in this chapter. . . . .	90
3.19	Sum rate vs. backhaul for downlink CoMP schemes and specific channels. . .	92
3.20	Monte Carlo simulation results for $M = K = 2$ in the downlink. . . . .	94
3.21	The best choice of BS cooperation scheme in downlink CoMP. . . . .	96
3.22	Impact of channel orthogonality and SNR on downlink CoMP schemes. . . .	98
3.23	Monte Carlo simulation results for $M = K = 3$ in the downlink. . . . .	99
4.1	Cellular setup considered for system level simulations. . . . .	102

---

4.2	Example of a highly asymmetric uplink interference scenario. . . . .	105
4.3	Clustering concepts compared through system level simulations. . . . .	107
4.4	Uplink SINRs obtainable through clustering and interference cancellation. . .	108
4.5	Downlink SINRs obtainable through clustering and interference cancellation.	109
4.6	User throughput distribution and average user throughput vs. backhaul for different cooperation sizes and strategies (uplink). . . . .	112
4.7	User throughput distribution and average user throughput vs. backhaul for different cooperation sizes and strategies (downlink). . . . .	113
5.1	Benefit of backhaul-aware CoMP scheduling in a scenario with $M = K = 2$ . .	125
E.1	Channel estimation and CSI feedback process. . . . .	164
E.2	Pilot structure in each PRB from the LTE downlink assumed in this appendix.	165
F.1	Illustration of adding one iteration to a DIS cooperation and decoding process.	170

# List of Tables

3.1	Relation between BS-UE assignment $\mathbf{m}$ and $\Psi$ for non-coop. DL transmission.	83
4.1	Parameters used for system level simulations. . . . .	103
4.2	Summary on Monte-Carlo simulation results for scenarios with $M = K = 3$ . .	116
4.3	Summary on system level simulation results. . . . .	116
5.1	Comparison of uplink BS cooperation schemes, considering practical aspects.	122
5.2	Comparison of downlink BS cooperation schemes, considering practical aspects.	123



# Abbreviations

3GPP	Third Generation Partnership Project
ARQ	Automatic Repeat Request
BC	Broadcast channel
bh.	backhaul
bpcu	bits per channel use
BS(s)	Base Station(s)
CDMA	Code Division Multiple Access
CEO	<i>here</i> : Chief Estimation Officer
CIF	Compressed Interference Forwarding
CoMP	Coordinated Multi-Point
CQI	Channel Quality Indicator
CSI	Channel State Information (or: <i>channel knowledge</i> )
CSIR	Channel State Information at the receiver side
CSIT	Channel State Information at the transmitter side
Ctr.	Centralized
DAS	Distributed Antenna System ( <i>uplink or downlink BS coop. scheme</i> )
Dec.	Decentralized
DIS	Distributed Interference Subtraction ( <i>uplink BS cooperation scheme</i> )
DPC	Dirty Paper Coding
DL	Downlink
FDD	Frequency Division Duplex
FDM	Frequency Division Multiplex
FFT	Fast Fourier Transform
GPS	Global Positioning System
HK	Han-Kobayashi scheme ( <i>here referring to a channel coding concept</i> )
IAP	Interference-Aware Precoding
IC	Interference Channel
ICI	Inter-Carrier Interference
I-DAS	Iterative DAS scheme ( <i>uplink BS cooperation scheme</i> )
I-DIS	Iterative DIS scheme ( <i>uplink BS cooperation scheme</i> )
i.i.d.	independently and identically distributed
IRC	Interference Rejection Combining
ISD	Inter-Site Distance
ISI	Inter-Symbol Interference
LTE	Long Term Evolution ( <i>evolution of mobile standards after UMTS</i> )

---

MAC	Multiple Access Channel
MAP	Maximum A posteriori Probability
mcp.c.	multi-cell power control
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
MMSE	Minimum Mean Square Error
MRC	Maximum Ratio Combining
MRT	Maximum Ratio Transmission
N-DAS	Network DAS ( <i>DAS scheme with decoding at a central network entity</i> )
OFDM	Orthogonal Frequency Division Multiplex
p.a. pwr.	per-antenna power constraint
pf. / ip. CSI	perfect / imperfect channel knowledge
PRB	Physical Resource Block ( <i>resource unit in LTE</i> )
QSC	Quantized Sequence based Cooperation ( <i>downlink BS coop. scheme</i> )
RRH	Remote Radio Head
SCER	Signal-to-Channel-Estimation-Error Ratio
SCM	Spatial Channel Model
SCO	SINR Constrained Optimization
SIC	Successive Interference Cancellation
SINR	Signal-to-Interference-and-Noise Ratio
SISO	Single Input Single Output
SNR	Signal-to-Noise Ratio
SOCP	Second Order Cone Program
SON	Self-Organizing Network
sum pwr.	sum power constraint
TDM	Time Division Multiplex
THP	Tomlinson Harashima Precoding
TS	Time-share ( <i>between no and infinite BS cooperation in downlink</i> )
TTI	Transmit Time Interval ( <i>corresponds to 1 ms in an LTE system</i> )
UE(s)	User Equipment or terminal(s)
UL	Uplink
UMC	Unquantized Message based Cooperation ( <i>downlink BS coop. scheme</i> )
UMTS	Universal Mobile Telecommunications System
ZF	Zero Forcing
WSSUS	Wide-Sense Stationary Uncorrelated Scattering

# Symbols

## Setup

$M, K$	Number of BSs and UEs, respectively
$\mathcal{M}, \mathcal{K}$	Sets of all involved BSs and UEs, respectively
$N_{\text{bs}}, N_{\text{mt}}$	Number of receive/transmit antennas per BS and UE, respectively
$N_{\text{BS}}, N_{\text{MT}}$	Overall no. receive/transmit ant. at BS and UE side, respectively
$\mathbf{d} = [d_1 \dots d_K]^T$	Normalized distance of UEs to their BSs
$\theta, \lambda_{m,k}$	Path loss exponent and path gain between BS $m$ and UE $k$
$\varphi_{AB}, \varphi_{Ab}, \dots$	Angles representing channel orthonogonality $\rightarrow$ see Section 3.1.3

## Transmission Model

$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K]$	Channel matrix
$\hat{\mathbf{H}}, \hat{\mathbf{H}}^{\text{UE}}, \hat{\mathbf{H}}^{\text{BS}}$	Estimated channel in UL and DL (UE and BS side), respectively
$\hat{\mathbf{E}}, \hat{\mathbf{E}}^{\text{UE}}, \hat{\mathbf{E}}^{\text{BS}}$	Channel estim. error in UL and DL (UE and BS side), respectively
$\mathbf{H}^e$	Power-reduced, effective channel due to imperfect CSI
$\mathbf{E}^e, \mathbf{E}^{e,\text{UE}}, \mathbf{E}^{e,\text{BS}}$	Effective channel estimation error in uplink and downlink (UE and BS side), respectively, uncorrelated from the estimated channel
$\mathbf{W}$	Precoding matrix employed in the downlink
$\Psi$	Diagonal scaling matrix connected to antenna selection and quantization in the downlink
$\mathbf{m} = [m_1 \dots m_K]^T$	Assignment of UEs to BSs
$\mathbf{\Pi}_{\text{ant}}$	Assignment of antennas to BSs
$\sigma^2, \sigma_E^2$	Background noise variance and channel estimation noise variance
$N_p, N_d$	Number of pilots per transmission block used in uplink and downlink, and number of quantization bits per channel coefficient used for CSI feedback
$t$	Symbol index (typically omitted in this work for brevity)
$\mathbf{s}^{[t]}$	Transmitted signals in channel access $t$
$\mathbf{y}^{[t]}, \mathbf{n}^{[t]}$	Received signals and noise in channel access $t$
$\mathbf{v}^{[t]}, \mathbf{v}^{\text{UE}}[t], \mathbf{v}^{\text{BS}}[t]$	Noise term in channel access $t$ caused by imperfect CSI in UL and DL (UE and BS-side), respectively
$S_k, Y_m, \bar{Y}_m$	Overall signal sequence transmitted by UE $k$ , and overall signal sequence (over all antennas) received by BS $m$ , before and after signal processing (only relevant in the uplink)
$N_{m,a}, N_k$	Noise sequence received at antenna $a$ of BS $m$ (uplink), and received by user $k$ (downlink)
$\mathbf{P} = \text{diag}(\mathbf{p})$	Uplink transmit power (or uplink transmit covariance)

$\mathbf{P}(\mathcal{F})$	Uplink transmit power connected to all messages in set $\mathcal{F}$
$\hat{\mathbf{P}}^{\max} = \text{diag}(\hat{\mathbf{p}}^{\max})$	Maximum transmit power (per user) in the uplink
$\check{\mathbf{P}}^{\max} = \text{diag}(\check{\mathbf{p}}^{\max})$	Maximum transmit power (per antenna, if applicable) in the uplink
$\hat{\mathbf{B}}^{\max}, \check{\mathbf{B}}^{\max}$	Total backhaul available in uplink and downlink, respectively
$\hat{\mathbf{B}}^{\text{dis}}, \hat{\mathbf{B}}^{\text{cif}}, \hat{\mathbf{B}}^{\text{das}}, \hat{\mathbf{B}}^{\text{net}}$	Extent of backhaul infrastructure invested into DIS, CIF and DAS schemes and network forwarding in the uplink, respectively
$\mathbf{C}$	Auxiliary variable used in the downlink, denoting the number of quantization bits used when providing messages to certain BSs
$\check{\mathbf{B}}(\mathbf{C})$	Backhaul infrastructure required in the downlink for a certain choice of auxiliary variable $\mathbf{C}$
$\beta$	Additional backhaul needed as compared to a non-cooperative system
$\beta(\mathbf{r}, \mathbf{B})$	Function returning the sum backhaul needed in addition to a non-cooperative system, given a rate tuple $\mathbf{r}$ and backhaul matrix $\mathbf{B}$

### Covariances and Quantization Noise

$\Phi^{\text{ss}}$	Downlink transmit covariance
$\Phi^{\text{hh}}$	Noise covariance connected to channel estimation errors
$\Phi_m^{\text{yy}}, \bar{\Phi}_m^{\text{yy}}$	Receive signal covariance at BS $m$ <i>before</i> and <i>after</i> signal processing
$\Phi_{k,m}^{\text{yy}}, \Phi_{k,m m'}^{\text{yy}}$	Receive signal covariance at BS $m$ connected to UE $k$ , and the same covariance <i>conditioned</i> on the receive signals at BS $m'$
$\bar{\Phi}_{m m'}^{\text{yy}}, \bar{\Phi}_{m m',\mathcal{M}'}^{\text{yy}}$	Receive signal covariance at BS $m$ after signal proc., conditioned on the receive signals at BS $m'$ , or conditioned on the receive signals at BS $m'$ and signals provided by BSs in $\mathcal{M}'$ to BS $m'$
$\Phi_{m \rightarrow m'}^{\text{qq}}$	Covariance of quantization noise introduced when forwarding receive signals from BS $m$ to BS $m'$

### Messages, Sequences and Functions

$N_{\text{sym}}$	Number of symbols transmitted successively in one block
$F, X$	Message (data bits) and sequence (of $N_{\text{sym}}$ symbols, assumed to be a Gaussian process), respectively
$\hat{\mathcal{F}}_{\text{all}}, \check{\mathcal{F}}_{\text{all}}$	Sets of all messages involved in UL or DL transmission, respectively
$\hat{\mathcal{F}}_{\text{all}^*}$	Set of all uplink messages <i>not</i> decoded by a central network entity
$\hat{\mathcal{F}}_k, \check{\mathcal{F}}_k$	Sets of all messages connected to UE $k$ , in UL or DL, respectively
$\hat{F}_k^{\mathcal{M}'}$	Uplink message originating from UE $k$ and decoded individually by all BSs in $\mathcal{M}'$
$\hat{F}_k^{\mathcal{M}', m \rightarrow \mathcal{M}''}$	Uplink message originating from UE $k$ , decoded individually by all BSs in $\mathcal{M}'$ , and DIS-forwarded by BS $m \in \mathcal{M}'$ to all BSs in $\mathcal{M}''$
$\hat{F}_k^{\mathcal{M}', m \nrightarrow \mathcal{M}''}$	Uplink message originating from UE $k$ , decoded individually by all BSs in $\mathcal{M}'$ , and CIF-forwarded by BS $m \in \mathcal{M}'$ to all BSs in $\mathcal{M}''$
$\mathcal{F}^{[m]}, \check{\mathcal{F}}^{[m]}$	Set of messages decoded by BS $m$ , and set of messages neither decoded by BS $m$ nor provided to BS $m$ by any other BS through DIS or CIF concepts
$\check{\mathcal{F}}^{[m]}, \mathcal{F}^{\nrightarrow [m]}$	Sets of messages provided to a BS $m$ through the DIS or CIF concept, respectively

$e(\cdot), d(\cdot)$	Encoding function, mapping a message $F$ to a sequence $X = e(F)$ , and corresponding decoding function
$\mathbf{g}(\cdot)$	Encoding function used for DPC in the downlink
$q(\cdot), s(\cdot)$	Quantization and Slepian-Wolf source coding function, respectively

### Terms connected to Uplink-Downlink Duality

$\hat{\Phi}_{\text{nn}}$	Noise covariance in the dual uplink
$\hat{\mathbf{P}} = \text{diag}(\hat{\mathbf{p}})$	Transmit powers in the dual uplink
$\mathcal{J}_1(k), \mathcal{J}_2(k)$	Sets of UEs causing interference or CSIT related noise in the DL
$\mathcal{J}_1^*(k), \mathcal{J}_2^*(k)$	Sets of UEs causing interference or CSIT related noise in the dual uplink (dual sets to $\mathcal{J}_1(k), \mathcal{J}_2(k)$ )

### Rates, Capacity Regions and Performance Regions

$\nu_F$	Rate connected to a message $F$
$\mathbf{r} = [r_1 \dots r_K]^T$	Rates connected to UEs
$\hat{\mathcal{R}}_\infty, \hat{\mathcal{R}}_0, \hat{\mathcal{R}}_0^{\text{fdm}}, \hat{\mathcal{R}}_0^{\text{hk}}$	Lower bounds on UL capacity regions for infinite BS coop. ( $\hat{\mathcal{R}}_\infty$ ), or no BS cooperation, assuming only one message per UE ( $\hat{\mathcal{R}}_0$ ), FDM ( $\hat{\mathcal{R}}_0^{\text{fdm}}$ ) or Han-Kobayashi concepts ( $\hat{\mathcal{R}}_0^{\text{hk}}$ )
$\check{\mathcal{R}}_\infty, \check{\mathcal{R}}_0, \check{\mathcal{R}}_0^{\text{hk}}$	Lower bounds on downlink capacity regions for infinite BS cooperation ( $\check{\mathcal{R}}_\infty$ ), or no BS cooperation, assuming only one message per UE ( $\check{\mathcal{R}}_0$ ), or Han-Kobayashi concepts ( $\check{\mathcal{R}}_0^{\text{hk}}$ )
$\hat{\mathcal{R}}^{\text{das,fdm}}(\hat{\mathbf{B}}^{\text{das}}, \hat{\mathbf{B}}^{\text{net}})$	Lower bound on the capacity region of DAS-enhanced FDM, using extents of backhaul $\hat{\mathbf{B}}^{\text{das}}$ and $\hat{\mathbf{B}}^{\text{net}}$
$\hat{\mathcal{R}}^{\text{coop}}(\hat{\mathbf{B}}^{\text{dis}}, \hat{\mathbf{B}}^{\text{cif}}, \hat{\mathbf{B}}^{\text{das}}, \hat{\mathbf{B}}^{\text{net}})$	Lower bound on cap. region of DIS/CIF/DAS schemes in UL
$\check{\mathcal{R}}_{\text{coop}}(\check{\mathbf{B}}^{\text{max}})$	Lower bound on cap. region for DAS/UMC/QSC schemes in DL
$\hat{\mathcal{Z}}^{\text{dis}}, \hat{\mathcal{Z}}^{\text{cif}}, \hat{\mathcal{Z}}^{\text{dasd}}, \hat{\mathcal{Z}}^{\text{dasc}}$	Performance regions connected to various schemes in the uplink, capturing both achievable rates as also the sum backhaul required in addition to a non-cooperative system
$\check{\mathcal{Z}}^{\text{das}}, \check{\mathcal{Z}}^{\text{ts}}, \check{\mathcal{Z}}^{\text{umc}}, \check{\mathcal{Z}}^{\text{qsc}}$	Performance regions connected to various schemes in the downlink
$\mathcal{R}_\beta^{\text{xy}}$	Constrained capacity region of scheme $xy$ , given sum-backhaul $\beta$
$f_s(\cdot), f_c(\cdot)$	Functions returning the maximum sum-rate of a capacity region, if the sum-rate itself or the common rate is maximized, respectively
$\alpha_k$	Weight applied to UE $k$ when performing weighted sum-rate maximization



# Chapter 1

## Introduction

### 1.1 Motivation

Mobile communication has gained significant importance in today's society. Just recently, the number of mobile phone users worldwide has surpassed 4 billion [WTI08], while the global annual mobile revenue is expected to top \$1 trillion in 2013 [TM08]. Beside conventional voice services, novel mobile applications such as location-based services, video conferencing or mobile gaming [Com09], and the demand for ubiquitous Internet connectivity have triggered an unprecedented growth of mobile data traffic. But though analysts predict this traffic to double annually in the next years [For09], mobile data revenues are merely expected to increase two-fold until 2013 [TM08], creating a severe challenge for mobile operators to respond to the demand for ubiquitous mobile bandwidth, while significantly reducing cost per bit.

These requirements can only be met if the *spectral efficiency* of mobile networks, i.e. the throughput achievable per bandwidth, is strongly increased. The denser, however, a network operator reuses licensed spectrum, the more the system performance becomes limited through inter-cell interference [GK00]. The recently finalized standard LTE Release 8 [Sch09, Erg09] partially addresses this problem by foreseeing multiple antennas at base station and terminal side [McC07], rendering so-called *multiple input - multiple output* (MIMO) techniques possible [FG98, Tel99, Tay04]. These enable spatial multiplexing (e.g. multiple data streams per communication link), array gain (as multiple antennas can coherently pick up or emit signal power), and interference mitigation (making use of the *spatial signature* of interference). As the number of deployable antennas is limited, e.g. through regulatory issues at the base station side, or form factor issues at the terminal side, other means are necessary to further increase spectral efficiency in the presence of inter-cell interference.

#### **Coordinated Multi-Point for Inter-Cell Interference Exploitation**

From information theory it is known that inter-cell interference can be seen as an opportunity, rather than a curse, if base stations *cooperatively* process signals [SSZ04]. Such techniques are often referred to as *virtual MIMO*, *network MIMO*, or, more recently, *Coordinated Multi-Point* (CoMP), and they are seen as a key technology of *LTE Advanced* [PDF<sup>+</sup>08, PA09]. Briefly, such schemes allow *interference exploitation* in the uplink through the joint detection of multiple terminals by cooperating base stations, or *interference avoidance* in the downlink, through the joint and coherent transmission from multiple base stations to multiple terminals. CoMP

schemes are also known to provide more *fairness*, i.e. a more homogeneous distribution of throughput over the area, an aspect so far insufficiently addressed in LTE Release 8. Whether CoMP can furthermore improve the *energy* or *cost* efficiency of cellular networks is a topic still under investigation. On one hand, such schemes reduce the transmit power required per transmitted bit, but increased complexity and other overhead might compensate for these efficiency gains. A comprehensive literature overview on CoMP is given in Appendix A.

### The Backhaul Bottleneck

Beside many challenges, one major issue connected to CoMP is the large network infrastructure required between cooperating base stations, typically referred to as *backhaul*. Even in current systems, the backhaul infrastructure tends to become the system bottleneck [Buc08, Chu08]. Consequently, the revenues of backhaul solution providers are expected to double in the next four years [Res09]. Introducing cooperation between base stations can easily lead to yet another n-fold increase of backhaul infrastructure [MF07b, MF07c] unless smart and backhaul-efficient cooperation techniques are employed. The focus of this work is hence on

- identifying scenarios in which CoMP is most beneficial, also taking into account the major impact of imperfect channel knowledge at base station and terminal side.
- analyzing a variety of CoMP concepts w.r.t. the achievable throughput/backhaul trade-off, and proposing general backhaul-efficient CoMP strategies for cellular systems.

### CoMP vs. Soft Handoff

Please note that CoMP is often wrongly equated with *soft handoff* concepts [VVGZ94, WL97] used in CDMA systems [Ass93]. Here, a cell-edge terminal is served by two or more base stations, such that it is instantaneously detected by the best base station in the uplink, yielding so-called *macro diversity*. In the downlink, the terminal receives individual transmissions from all involved base stations and can jointly exploit them through *maximum ratio combining* (MRC) [MLG99]. In both cases, multiple resources have to be reserved for this terminal, leading to an effective loss of spectral efficiency. Furthermore, soft handoff does *not* aim at exploiting spatial multiplexing gain or combating inter-cell interference, but is solely targeted towards improving the performance of handoff processes between cells.

## 1.2 Contribution of this Work

### Information-theoretic Analysis of the Throughput/Backhaul Trade-Off

In this work, the throughput/backhaul trade-off of various CoMP strategies is investigated, also taking into account the detrimental impact of imperfect channel knowledge. The topic is observed from an information-theoretic point of view, where existing work has not sufficiently captured the many degrees of freedom of backhaul-efficient CoMP or provided conclusive answers yet. The research in this work is initially based on reasonably dimensioned and detailed CoMP scenarios that are still analytically tractable, while yielding a more meaningful insight into the topic than the models of other authors. The results are then complemented with a system-level perspective on backhaul-efficient CoMP, as well as a discussion on practical issues connected to the considered schemes.



## Major Conceptual and Theoretical Contributions

Besides delivering a comprehensive overview on the issue of backhaul-aware CoMP, this thesis also provides new methodology for the characterization of the downlink capacity region under no, infinite or partial base station cooperation, and under imperfect channel knowledge at base station and terminal side. A major theoretical contribution is the generalization of uplink/downlink duality to these aspects [MF09a], as well as the introduction of the concept of *performance regions* [MF08e]. The latter enable to capture both the achievable rates of terminals under certain cooperation schemes, as well as the required backhaul.

## 1.3 Related Work

To the best of our knowledge, there are only few research groups beside the Vodafone Chair that have worked on the topic of backhaul-efficient CoMP from an information-theoretic point of view:

**Amichai Sanderovich, Oren Somekh, Osvaldo Simeone, Shlomo Shamai (Shitz), Benjamin M. Zaidel and Vincent Poor** have written a multitude of publications connected to backhaul-constrained CoMP in uplink and downlink. Their research is mainly based on simplified cellular scenarios, such as one- or two-dimensional Wyner models [Wyn94], where intra- and inter-cell signal propagation is characterized through very few parameters. This facilitates the derivation of analytical expressions, through which for example asymptotic throughput/backhaul trade-offs for an infinite transmit power, number of cells etc. can be investigated. The key findings of the stated authors are summarized as follows:

The authors initially investigated uplink CoMP in [SSSK05, SSSP06, SSS09], observing a two-antenna transmitter and two receiving base stations, which independently quantize and forward their received signals to a central processing unit in the network. The authors point out that large gains in the throughput/backhaul trade-off can be obtained if quantization schemes are used that exploit the signal correlation between different base stations. Observations were extended to an arbitrary number of base stations with symmetric inter-cell interference in [SSS07a, SSS<sup>+</sup>07b, SSS<sup>+</sup>08a, SSS<sup>+</sup>08b]. It was shown that the throughput/backhaul trade-off can be further improved if partial decoding already takes place at the base stations, hence prior to cooperation. For a slightly modified setup with asymmetric interference links, the authors have introduced a set of base station cooperation concepts in [SSPS08b, SSPSO9b]. These include the possibility that base stations decode the transmission of an assigned terminal and then forward the decoded bits (or any representation thereof) to another base station for interference cancellation, similar to concepts discussed in [KF07, MF08e, KF08].

Regarding the cellular downlink, the authors have also considered a circular Wyner model with simplified, asymmetrical interference in [SSPS07, SSS<sup>+</sup>07b, SSSP08, SSS<sup>+</sup>08a, SSS<sup>+</sup>08b, SSPSO8a, SSPSO9a]. The authors compare cooperation strategies where each base station either performs local encoding (possibly with knowledge on the encoding function of a set of adjacent base stations), where a central network entity performs the encoding for the transmissions targeted to all terminals and forwards quantized signals to the base stations, or a combination of both. They conclude that local encoding approaches are only superior under strongly constrained backhaul and large SINR, and otherwise inferior to centralized approaches, while mixed strategies are not beneficial at all.

**Aitor del Coso** and **Sebastian Simoens** have worked on distributed compression of received signals for cooperation in a cellular uplink [dCS08,dS08]. Their model foresees decoding to take place both at a centralized network entity, or by one of the base stations involved. Basically, their work is a generalization of distributed compression and source coding schemes introduced in [SSSK05,SSSP06] to scenarios with an arbitrary number of antennas per base station, which is essential for observing achievable throughput/backhaul trade-offs for MIMO channels under fast fading realizations. The cited work hence provides a fundamental mathematical basis for the models derived in this thesis.

Recently, **I-Hsiang Wang** and **David Tse** have started investigating interfering transmissions under partial receiver-side cooperation [WT09], but have focused on the observation of strong interference cases and regimes of asymptotically large signal-to-noise ratio, which is probably of minor value for the practical usage of CoMP.

Note that some authors consider base station cooperation to take place over the same wireless resource as the communication between terminals and base stations [HM06,PV09], which, however, is an entirely different scenario than the one considered in this thesis.

## 1.4 Structure of this Thesis

The thesis is organized as follows:

In Chapter 2, the transmission models considered for uplink and downlink CoMP are introduced, and inner bounds on *capacity regions* for a non-cooperative, partially cooperative (i.e. backhaul-constrained), or fully cooperative system under imperfect channel knowledge at the transmitter and receiver side are derived. Furthermore, the before mentioned concept of *performance regions* is introduced.

In Chapter 3, the general models stated before are used for the observation of small cooperation scenarios that are still analytically and numerically tractable, while yielding a valuable insight into the degrees of freedom of CoMP. General gains expectable through CoMP in uplink and downlink are observed, and the throughput/backhaul trade-off achievable with the introduced cooperation concepts is evaluated for various scenarios.

Observations are extended to large cellular systems in Chapter 4, where *clustering* and *resource partitioning* concepts are introduced in order to break down such systems into the cooperation scenarios treated before. It is shown that the typical structure of cellular systems allows large portions of CoMP gains to be obtained at a reasonable investment into backhaul.

After a comprehensive discussion on the implications of the models and key findings of this work on practical cellular systems in Chapter 5, the work is concluded in Chapter 6.

## 1.5 Notation

The following notation is used throughout the work:

- Capital, italic letters (e.g.  $X, Y$ ) refer to *sequences* of transmitted or received symbols.
- Capital, bold letters (e.g.  $\mathbf{H}^e, \mathbf{C}$ ) denote matrices (superscripts distinguish different matrices), where single column vectors are denoted through the corresponding lower-case letter and the column index, e.g.  $\mathbf{h}_k^e, \mathbf{c}_k$ . A single element in the  $i$ th row and  $j$ th column of the matrix is addressed as  $h_{i,j}^e, c_{i,j}$ , respectively. A notation such as  $\mathbf{h}_{m,k}^e$  or  $\Phi_m^{\text{nn}}$  can refer to a sub-part of a matrix, which will be explained explicitly where necessary. Operator  $\text{vec}(\cdot)$  stacks all columns of a matrix into one long column vector.
- $\mathbf{A} \succeq 0$  denotes positive semidefiniteness,  $\mathbf{A} \succeq \mathbf{B}$  states that  $\mathbf{A} - \mathbf{B}$  is positive semidefinite, and  $\mathbf{A} > \mathbf{B}$  denotes element-wise inequality.
- Calligraphic letters (e.g.  $\mathcal{M}, \mathcal{F}$ ) refer to sets,  $\emptyset$  refers to the empty set, and e.g.  $|\mathcal{M}|$  denotes the size, or cardinality, of a set.
- The sets of real, complex and integer numbers are denoted as  $\mathbb{R}, \mathbb{C}$ , and  $\mathbb{N}$ , respectively.
- Operator  $\Delta(\cdot)$  is used on symmetric matrices and sets all off-diagonal values to zero, while operator  $\mathbf{M} = \text{diag}(\mathbf{m}), \mathbf{m} = \text{diag}(\mathbf{M})$  returns a symmetric matrix  $\mathbf{M}$  with diagonal elements taken from vector  $\mathbf{m}$ , or extracts the diagonal  $\mathbf{m}$  from a given matrix  $\mathbf{M}$ , depending on the operand, as known from MATLAB.
- Operators  $H(\cdot)$  and  $h(\cdot)$  denote entropy and differential entropy, respectively, and  $I(X; Y)$  denotes the mutual information between  $X$  and  $Y$ .
- Expressions  $(\cdot)^T$  and  $(\cdot)^H$  denote matrix and Hermitian transpose, respectively.
- Various variables are used with an accent (i.e.  $\hat{X}, \check{X}$ ) to indicate their connection to the uplink or downlink, respectively.
- The operator  $\cup$  denotes a convex hull operation, and  $E_{xy}\{\cdot\}$  denotes the expectation value of the term in parentheses over many realizations of  $xy$ .
- $\mathbf{I}$  denotes the identity matrix, and  $\mathbf{0}_{[i \times j]}, \mathbf{1}_{[i \times j]}$  denote matrices of size  $i \times j$ , filled with zeros or ones, respectively.
- The notation  $\mathbf{x} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{m}, \Phi)$  states that  $\mathbf{x}$  is a vector of complex Gaussian random variables with mean  $E\{\mathbf{x}\} = \mathbf{m}$  and covariance  $E\{\mathbf{x}\mathbf{x}^H\} = \Phi$ .
- $\text{tr}\{\mathbf{A}\}$  and  $|\mathbf{A}|$  denote matrix trace and determinant, respectively.