

Vinay Suryaprakash

Modeling and Analysis of Radio Access Networks using Spatial
Point Processes

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Vinay Suryaprakash

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Modeling and Analysis of Radio Access Networks using Spatial Point Processes

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Abstract

The popularity of smart phones and the rapid expansion of mobile data services have made catering to user demands, of high data rates and ubiquitous coverage, more daunting than ever before. Efforts to meet these challenges have resulted in the deployment of increasingly dense and heterogeneous networks. This thesis provides models using spatial point processes to help understand, analyze, and optimize these networks.

This thesis begins by providing simple extensions, in the form of increased correlation between users and base stations, to existing work on homogeneous networks using stationary Poisson processes. These efforts result in expressions for the interference, the probability of coverage, and spectral efficiency (or spatially averaged rate). The expressions are utilized to calculate the energy consumed by the network as well as compare two commonly used energy management strategies, namely, sleep modes and bandwidth variation. These investigations reveal that the amount of energy consumed is highly dependent on the network load, and that sleep modes are more effective in saving energy than bandwidth variation. Then, staying with homogeneous networks, this thesis furnishes a model that helps compute the deployment cost of a network with k network components, while ensuring that users' demands are met. As a special case, a three layer network consisting of users, base stations, and backhaul nodes is considered. The expression for deployment cost in this case is found, and it is followed by a numerical evaluation, which shows that there exists a backhaul intensity which can minimize the deployment cost while guaranteeing users' demands.

The focus of this thesis is subsequently shifted to heterogeneous networks, where it provides an alternative to the most popular method of modeling heterogeneous networks, viz. the superposition of independent, stationary Poisson processes. The model proposed here uses a stationary Poisson cluster process in which micro base stations are clustered around macro base stations. An expression which provides the complete description of the interference in such a network is found. The utility of this find is demonstrated by finding expressions for the probability of coverage and spectral efficiency. Though the expressions found are easy to evaluate numerically, they are rather cumbersome. Therefore, in an effort to describe them more succinctly, the asymptotic behavior of the interference is examined using

an estimator. This results in a theorem which states that the distribution of the interference is asymptotically Gaussian. Following which, the expressions for the mean and the variance of the interference are obtained. The asymptotic convergence of the interference as well as the expressions for its mean and variance are verified by using Monte-Carlo simulations. These simulations corroborate the accuracy of these findings and also reveal that, in most optimization problems that deal with dense networks, the sole use of the mean of the interference should suffice.

After which, the focus of this thesis is briefly shifted back to homogeneous networks, where a stationary Neyman-Scott process is used to model a network in which users are clustered around base stations. As in the previous cases, an expression for a complete description of the interference, along with expressions for the probability of coverage and spectral efficiency, is derived. These expressions are also numerically evaluated. Once again, in an effort to make these expressions more tractable, the asymptotic behavior of the interference is examined. Interestingly, this results in a theorem which also states that the distribution of the interference is asymptotically Gaussian. Thereby, making the expressions, which were previously found, more convenient to use.

Lastly, this thesis establishes a model to analyze the deployment cost of a network consisting of two backhaul technologies (namely, microwave and fiber optic backhaul), two types of base stations (i.e., macro and micro base stations), and users. An expression for the average cost of deploying a backhaul node in a network, which can cater to users' demands effectively, is found. This is later used to find the total deployment cost of the network. Numerical evaluations of these expressions show that, as in the previous cost model, there exist a range of backhaul intensities that can minimize the deployment cost of a network while satisfying the given constraints.

Zusammenfassung

Die Popularität von Smartphones und die rasche Entwicklung mobiler Datendienste haben den Bedarf nach hohen Datenraten und einer flächendeckenden Netzabdeckung stark ansteigen lassen. Um diese Herausforderungen zu erfüllen, werden immer dichtere und heterogene Funknetzwerke eingesetzt. Diese Doktorarbeit liefert räumliche Punktprozessmodelle, die dazu dienen, diese Netzwerke zu verstehen, zu analysieren und zu optimieren.

Diese Doktorarbeit beginnt mit einfachen Erweiterungen, in Gestalt von verbesserter Korrelation zwischen Nutzer und Basisstationen, zu bestehenden Veröffentlichungen an stationäre Poisson Prozessmodelle für homogene Netzwerke. Diese Bemühungen ergeben mathematische Ausdrücke für die Interferenz, die Abdeckungswahrscheinlichkeit und die spektrale Effizienz (oder räumlich gemittelte Datenrate). Anschließend werden diese Ausdrücke verwendet, um den Energieverbrauch eines Netzwerks zu berechnen und die zwei am häufigsten eingesetzten Strategien zum Energiesparen (Sleep Modes und Variation der Bandbreite) zu vergleichen. Diese Untersuchungen zeigen, dass die Menge der verbrauchten Energie in hohem Maße von der Netzlast abhängig ist und dass Sleep Modes bezogen auf das Potenzial zur Energieeinsparung effektiver als eine Variation der Bandbreite sind. Danach liefert diese Doktorarbeit ein Modell, das die Berechnung der Installationskosten für ein Netzwerk mit k Netzwerkkomponenten unter Berücksichtigung des Verkehrsaufkommens, ermöglicht. Als Sonderfall wird ein dreischichtiges Netzwerk, bestehend aus Nutzern, Basisstationen und Backhaul Nodes, betrachtet. Es kann ebenfalls ein analytischer Ausdruck für die Einsatzkosten gefunden werden. Eine anschließende numerische Auswertung unternommen, die zeigt, dass eine bestimmte Dichte an Backhaul-Knoten existiert, bei der sowohl der Nutzerbedarf befriedigt als auch die Installationskosten des Netzwerks minimiert werden können.

Der Fokus dieser Doktorarbeit wird anschließend auf den Bereich heterogener Netzwerke verschoben. In der Literatur wird häufig eine Superposition unabhängiger, stationärer Poisson-Prozess zur Modellierung heterogener Netzwerke genutzt. Als Alternative dazu wird in dieser Arbeit ein Modell vorgeschlagen, das einen stationären Poisson-Clusterprozess verwendet, um die räumliche Gruppierung von Mikro-Basisstationen um Makro-Basisstationen

zu beschreiben. Des Weiteren wird ein Ausdruck zur vollständigen Beschreibung der Interferenz gefunden. Dass von diesem Ausdruck weitere wichtige Größen wie Abdeckungswahrscheinlichkeit und spektrale Effizienz abgeleitet werden können, zeigt die Nützlichkeit der vollständigen Interferenzbeschreibung. Diese genannten Ausdrücke und Größen sind zwar numerisch berechenbar, allerdings ist ihre Form unhandlich. Daher wird in dem Bemühen, die Ausdrücke kurz und bündig zu beschreiben, das asymptotische Verhalten des Schätzers der Interferenz untersucht. Diese Untersuchung führt zu einem Theorem, das besagt, dass die Interferenz asymptotisch Gauß-verteilt ist. Danach werden Ausdrücke für den Mittelwert und die Varianz der Interferenz ermittelt und die asymptotische Konvergenz sowie die Ausdrücke für den Mittelwert und die Varianz der Interferenz durch Monte-Carlo Simulationen verifiziert. Diese Simulationen bestätigen die Genauigkeit der Ergebnisse und zeigen, dass in vielen Optimierungsproblemen, die dichte Funknetze betrachten die Verwendung des Mittelwerts der Interferenz ausreichen sollte.

Anschließend wird der Fokus dieser Doktorarbeit kurzzeitig zurück auf homogene Netztopologien verschoben. Es wird ein stationärer Neyman-Scott Prozess verwendet, um ein Netzwerk, in denen die Nutzer um die Basisstationen gruppiert sind, zu modellieren. Wie in den vorhergehenden Fällen werden Ausdrücke für eine vollständige Beschreibung der Interferenz, für die Abdeckungswahrscheinlichkeit und die spektrale Effizienz abgeleitet. Diese Ausdrücke werden auch numerisch und das asymptotische Verhalten des Schätzers der Interferenz untersucht. Interessanterweise führt die Untersuchung zu einem Theorem, das ebenfalls besagt, dass die Verteilung der Interferenz asymptotisch Gauß ist. Dadurch werden die Ausdrücke, die zuvor gefunden wurden, vereinfacht und somit sind diese einfacher anwendbar.

Schließlich wird in dieser Doktorarbeit ein Modell entwickelt, um die Installationskosten eines Netzwerks, das aus zwei Backhaul-Technologien (Richtfunk- und Glasfaseranbindungen), zwei Typen von Basisstationen (Makro- und Mikro-Basisstationen) und Nutzern besteht, analysieren. Ein Ausdruck für die durchschnittlichen Installationskosten eines Backhaulknotens in einem Netzwerk, das auf das Verkehrsaufkommen wirksam eingehen kann, wird ermittelt. Dieser wird später verwendet, um die gesamten Installationskosten des Netzwerks zu bestimmen. Numerische Auswertungen dieser Ausdrücke zeigen, dass, wie in dem vorherigen Kostenmodell, bestimmte Dichten an Backhaulknoten existieren, die die Installationskosten eines Netzwerks minimieren können.

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“Knowledge is in the end based on acknowledgment.”

– Ludwig Wittgenstein

This thesis marks the culmination of my formal education which, as Wittgenstein points out, would not be possible without acknowledgments. Though Wittgenstein used the word “acknowledgment” to mean an internal affirmation, acceptance, or realization, a recognition of external influences that aid us in our efforts towards such a goal is essential. Key contributions from several individuals are the reasons for which I am able to see this work to completion.

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Moving to a new country is a challenging proposition, and it is even more so, if one is unfamiliar with the language spoken at their destination of choice. This is the situation I found myself in a few years ago, and my years pursuing a doctorate would not have been so memorable and enjoyable were it not for my friends and colleagues. My heartfelt thanks go out to my friends – Albrecht, André, David, Fabian, Henrik, Michael, Patrick, Rohit,

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1. Introduction

“He who seeks for methods without having a definite problem in mind seeks for the most part in vain.”

– David Hilbert

The surging popularity of mobile phones and an expansion in mobile wireless services, [ITUa], [ITUb], have increased demands for ubiquitous coverage in conjunction with high data rates resulting in progressively denser networks. Dense networks are accompanied by increases in the capital invested as well as the energy consumed (see [PWC]). It is, therefore, imperative that these networks be analyzed and optimized to ensure customer satisfaction and a reduction in expenses. A network can be made denser either by deploying more base stations of the same type, or by deploying base stations of different types. Networks with base stations of a single type are known as homogeneous networks and networks with more than one type of base station are known as heterogeneous networks. Traditional methods of analysis and optimization of homogeneous networks, for both theory and simulation based investigations, have modeled base station deployments as a hexagonal grid, where the base stations are located at the vertices of a hexagon (see e.g., [RFF09]). Similarly, heterogeneous networks are modeled with larger (macro) base stations located at the vertices of a hexagonal grid with smaller (micro, pico, or femto) base stations distributed randomly within the coverage area of the macro base stations (see e.g., [FRF09, RF10, RFMF10]).

However, real world deployments tend to be far more irregular owing to geographic and man-made structures like mountains, lakes, rivers, streets, historical monuments, etc. A solution that is sometimes used to introduce a measure of irregularity in the deployment is to randomly omit the placement of a base station at some of the vertices of the hexagonal grid. Such methods are limited in their ability and are unable to analyze deployments covering large areas, such as states or countries, based solely on the number of users and base stations in the area without having to be concerned with a specific type of deployment such as a Manhattan grid, or a hexagonal grid, etc. (see [3GP]). Another important shortcoming, in the case of simulation based methods of analysis and optimization, is the fact that most

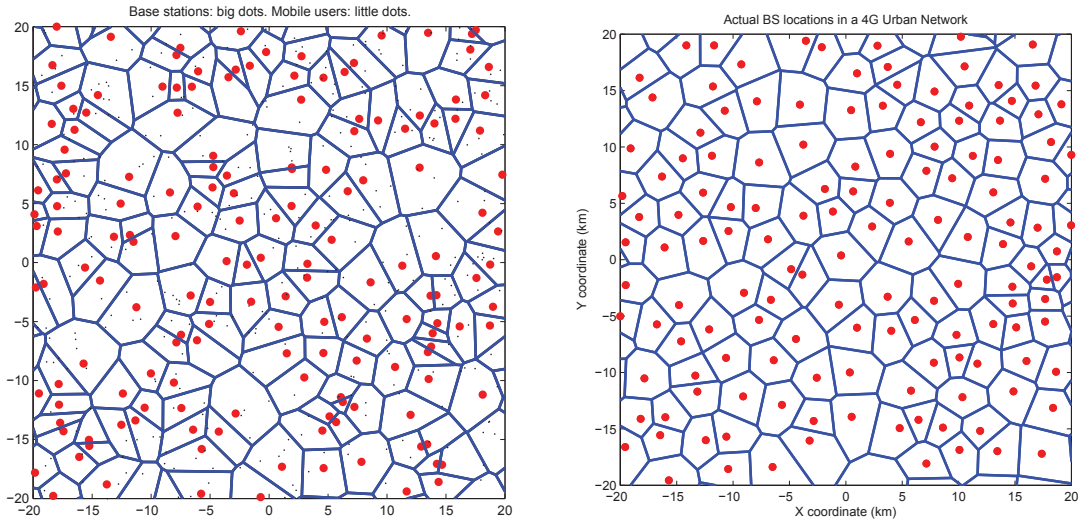
simulations are carried out on proprietary simulators which hinder comparisons between the results obtained. Therefore, obtaining a theoretic framework which can reflect the irregularity of modern networks, while acting as a benchmark against which simulations from different proprietary simulators can be compared, will prove extremely beneficial. This thesis attempts to provide such frameworks by modeling base stations and users as points of spatial point processes which are at the heart of a subject of study called stochastic geometry.

1.1. Stochastic Geometry: A brief overview

The modern theory of stochastic geometry started in the 1960's and is an offshoot of geometrical probability which was first studied in the 19th century. It is an area of mathematical research which seeks to provide models and methods to analyze complicated geometric patterns that occur in many areas of science and technology such as geology, biology, stereology, material sciences, etc. An attempt to highlight some of the applications of this subject was first seen in the German monograph titled "*Stochastische Geometrie: eine Einführung*", [Sto83]. Stochastic geometry was first utilized by Baccelli et al. in the late 1990's to analyze wired telecommunication networks (see [BZ99]). Baccelli et al. also published the first paper dealing with stochastic geometric models for wireless networks in the early 2000's (see [BB01]), followed by more detailed monographs [BB09b, BB09a] in 2009. The utility of this mathematical tool for wireless network analysis, as evidenced by recent works such as [HAB⁺09, AGH⁺10, ABG10, BBM10, BBM11, BK12, DGBA12], etc., has turned this into a burgeoning area of research.

The work done by Andrews et al. in [ABG10], as seen in figures 1.1(a) and 1.1(b), shows that such models can reflect real world deployments closely. Their work also demonstrates the ability of these models to be able to compute network performance indicators (such as probability of coverage, spectral efficiency, etc.) accurately, expediently, and with ease. These works also illustrate the usefulness of system level models and show that they can save a lot of computational effort while providing a benchmark against which other system level simulations can be compared.

This thesis extends upon the work done for homogeneous networks in some of the papers listed above, while exploring alternate models for heterogeneous networks. Before delving into the main contributions of this work, since these models are intended for use in other optimization problems, it is especially important to elaborate upon the timescales at which the system level analysis is carried out.



(a) Poisson distributed base stations and mobiles, where each mobile is associated with its nearest base station. The cell boundaries form a Voronoi tessellation. (b) A 40×40 km view of a current base station deployment by a major service provider in a relatively flat urban area, with cell boundaries corresponding to a Voronoi tessellation.

Fig. 1.1.: A comparison of a Poisson process model with a real world deployment from [ABG10].

1.2. System Level Analysis and its Timescales

The problems of interest, as far as this thesis is concerned, are ones related to network energy consumption, energy minimization, and the network's capital expenditure. These problems require taking the whole network or complete system into consideration rather than focusing on individual components. Thereby, making system level analysis more pertinent than analyses that focus on individual links between users and base stations. Since modern mobile networks are extremely complicated, it is common practice to analyze the network on different timescales. The timescales for system analysis can be broadly classified into three categories. The first corresponds to analyzing network behavior on a millisecond to second basis. The second corresponds to analyzing network behavior on the order of minutes to hours, and the last corresponds to analysis over the course of days to weeks. Network analysis and optimization at the smallest timescale deals mostly with problems related to radio resource allocation, and adapting transmissions to the wireless channel and instantaneous network load (see e.g., [Zan97, LZ06, IF10, IF11]). Analyses at the intermediate timescale deal with an examination of network management techniques which adapt the network to daily (or hourly) variations in traffic, such as those experienced during peak hours

and non-peak hours. Lastly, analyses at the largest timescale are best suited for new network roll-outs or network expansion based on requirements related to customer satisfaction.

The work presented in the subsequent chapters of this thesis deals with models that are best suited for use within the two larger timescales mentioned above. However, it is important to note that the models presented here include descriptions of the statistical behavior of the network during the smallest timescale.

1.3. Contributions and Organization

This thesis was motivated by the need to find a mathematical framework which can subsequently be used as a constraint in optimization problems related to energy efficiency and deployment costs. The framework establishes a relationship between the number of users, number of base stations, system level parameters (such as transmit power, thresholds, pathloss functions, pathloss exponents, etc.), and indicators of system performance such as probability of coverage or spectral efficiency in an area. Heeding David Hilbert's cautionary remark¹, we explore methods using point processes with the intention of establishing the relationship described above. The work done in this thesis can broadly be categorized in to two parts. One deals with modeling homogeneous networks using a homogeneous Poisson process, which is detailed in Chapter 2, and the other deals with models for homogeneous as well as heterogeneous networks using a stationary Poisson cluster process, which is detailed in Chapter 3. Since the mathematical definitions and assumptions required in Chapters 2 and 3 are different, they are designed to be self contained (for the most part). This, however, does not imply that the two chapters are completely independent of one another and it should be noted that the last part of Chapter 3 also utilizes some concepts from Chapter 2.

Chapter 2 uses a homogeneous Poisson process to model a homogeneous network. Section 2.2 of this chapter improves upon a model developed in [ABG10], and calculates the energy consumed by a mobile network in which the average demands of the users are met. This model is then used to compare the effectiveness of two different energy management strategies, namely sleep modes and bandwidth variation. Then, Section 2.3 develops a scheme (inspired by [BZ99]) to help analyze the deployment costs of a homogeneous network that, apart from users and base stations, consists of many different network components such as network concentrators (or backhaul nodes), Evolved Packet Core (EPC), etc. An expression to calculate the average cost of deploying a backhaul node in a network which meets the

¹Quoted at the beginning of this chapter.

users' average rate requirement is derived along with a generalization that allows the computation of the cost of deploying a particular node at the i^{th} component layer of a network which consists of ' k ' different network components. The relationship between spectral efficiency and the system parameters, established in the first half of the chapter, is used as a constraint that determines the cost of a base station's deployment. Using numerical examples, the cost calculation framework is used to show that there exists an optimal backhaul node intensity that can minimize deployment costs while ensuring that the users are provided the (average) rate demanded by them.

Chapter 3 uses a stationary Poisson cluster process as a basis for modeling different networks. This chapter deals with three different types of networks in Sections 3.2 – 3.4. The first, i.e., Section 3.2, uses a stationary Poisson cluster process to model a heterogeneous network consisting of two types of base stations (i.e., macro and micro base stations). It details a framework that could be used as an alternative to the framework developed in [DGBA12]. The first half of this section derives an expression for interference in heterogeneous networks and uses it to compute the probability of coverage in such networks. In the second half of Section 3.2, the asymptotic behavior of the interference is examined in an effort to describe it more succinctly, so that it can facilitate easier reuse in optimization problems. Then, in an effort to improve upon the models in Chapter 2, Section 3.3 uses a stationary Poisson cluster process to model a homogeneous network in which users are clustered around base stations. This section, similar to Section 3.2, derives an expression for interference in such a network and observes its asymptotic behavior. Lastly, Section 3.4 (like the second half of Chapter 2) deals with a framework to help analyze the deployment cost of a network which consists of users (modeled by a Poisson process), heterogeneous base stations (modeled using a Poisson cluster process), and backhaul nodes (modeled using a mixed Poisson process).

Finally, Chapter 4 draws conclusions from the work done in this thesis and presents the outlook for future research in this area.

