Beiträge aus der Informationstechnik

Mobile Nachrichtenübertragung
Nr. 83

Alexandros Pollakis

Towards self-organised synchronisation in networks of phase-locked loops

VOGT

Dresden 2016

Bibliografische Information der Deutschen Nationalbibliothek
Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.dnb.de abrufbar.

Bibliographic Information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at http://dnb.dnb.de.

Zugl.: Dresden, Techn. Univ., Diss., 2016

Die vorliegende Arbeit stimmt mit dem Original der Dissertation „Towards self-organised synchronisation in networks of phase-locked loops“ von Alexandros Pollakis überein.

Gesetzt vom Autor

ISBN 978-3-95947-007-0

Jörg Vogt Verlag
Niederwaldstr. 36
01277 Dresden
Germany

Phone: +49-(0)351-31403921
Telefax: +49-(0)351-31403918
e-mail: info@vogtverlag.de
Internet : www.vogtverlag.de

TECHNISCHE UNIVERSITÄT DRESDEN

Towards self-organised SYNCHRONISATION in networks of phase-locked loops

ALEXANDROS POLLAKIS

der Fakultät Elektrotechnik und Informationstechnik
der Technischen Universität Dresden

GENEHMIGTE DISSERTATION
zur Erlangung des akademischen Grades
Doktoringenieur
(Dr.-Ing.)

TECHNISCHE UNIVERSITÄT DRESDEN

Eingereicht am
14. Juni 2016

Verteidigt am
22. September 2016

Vorsitzender
Prof. Dr.-Ing. habil. Udo Jörges

Erster Gutachter
Prof. Dr.-Ing. Dr. h.c. Gerhard Fettweis

Zweiter Gutachter
Prof. Dr.-Ing. Gerd Ascheid

Abstract

In modern communication systems it is crucial for functionality that multiple components perform their operations in synchrony. In such systems a common time reference is key to provide the required coordination in time. For instance, large-scale antenna arrays require phase coherent carrier signals at each antenna unit that can be spatially distributed. In this thesis, we address the challenge of how synchronisation can be achieved in such systems. We show that networks of mutually coupled phase-locked loops can self-organise to provide a frequency-synchronous clock reference in spatially distributed systems of large scale. We develop a non-linear phase description of individual and coupled phase-locked loops that takes into account the characteristics of the filters and the delayed signal transmissions. Our phase model permits analytical expressions for the collective frequencies of the synchronised states as well as an analysis of the states' stability including the time scale of synchronisation. In particular, we find that the transmission delay between coupled phase-locked loops has profound effect on the synchronisation dynamics of the network. The delays have impact on the collective frequency of the synchronised states and can influence stability. Additionally, we show that the presence of filters in the system introduces stability transitions that are not found in systems without filtering. To test our theoretical results against real systems, we design and carry out experiments using networks of digital phase-locked loops. We show that our phase model can predict the existence and stability of synchronised states, as well as their characteristics. Our results demonstrate that networks of mutually delay-coupled phase-locked loops can provide a robust self-organised synchronous time reference for modern communication systems.

Zusammenfassung

In modernen Kommunikationssystemen ist Synchronität der Einzelkomponenten von wesentlicher Bedeutung für die Funktionalität des Gesamtsystems. Eine gemeinsame Zeitreferenz wird benötigt um die erforderliche zeitliche Koordination zu realisieren. Beispielsweise werden in großen Array-Antennen phasenkohärente Trägersignal an den oftmals räumlich verteilten Antenneneinheiten benötigt. Diese Dissertation adressiert die Fragestellung wie Synchronität systemweit über eine Vielzahl von räumlich verteilten Einzelkomponenten ermöglicht werden kann. Wir zeigen, dass Netzwerke miteinander gekoppelter elektronischer Phasenregelschleifen geordnete Zustände gleicher Frequenz eingehen können. Diese Zustände stellen sich in eigenständiger Art und Weise ein und können genutzt werden um räumlich ausgedehnte Systeme mit einer gemeinsamen Zeitreferenz zu versorgen. Zur analytische Betrachtung entwickeln wir ein (nichtlineares) Phasenmodell für einzelne und miteinander gekoppelte Phasenregelschleifen. In unserem Modell werden berücksichtigen wir sowohl die Eigenschaften der Filter als auch die auftretenden Übertragungsverzögerungen der Signale. Basierend auf unserem Phasenmodell erhalten wir analytische Ausdrücke für die kollektiven Frequenzen der synchronen Zustände. Zudem erlaubt es die Analyse deren Stabilität und die Ermittlung einer Synchronisationszeit/-dauer als zeitlichen Maßstab der Synchronisation. Wir stellen insbesondere fest, dass Übertragungsverzögerungen der Signale zwischen gekoppelten Phasenregelschleifen die Synchronität und deren Eigenschaften bezüglich kollektiver Frequenz und Stabilität maßgeblich beeinflussen. Zudem stellen wir fest, dass durch die in den Phasenregelschleifen enthaltenen Filter neue Stabilitätsübergänge auftauchen, welche in Systemen ohne Filter nicht auftreten. Um unsere Theory zu überprüfen führen wir Experimente durch, in welchen wir Netzw-

erke bestehend aus digitalen Phasenregelschleifen untersuchen. Wir zeigen, dass es anhand unseres Phasenmodells möglich ist das Synchronisationsverhalten realer Systeme quantitativ vorherzusagen. Im Speziellen weisen wir die Existenz der eingeführten synchronen Zustände nach, untersuchen deren Stabilität und deren charakteristischen Eigenschaften wie z.B. die kollektiven Frequenz und Synchronisationszeit. Die Ergebnisse unserer Arbeit zeigen, dass Netzwerke bestehend aus miteinander gekoppelten elektronischer Phasenregelschleifen selbstorganisiert robuste und frequenzsynchrone Zustände einnehmen können. Solche Netzwerke können als Zeitgeber für moderne Kommunikationssysteme genutzt werden, insbesondere wenn diese aus einer Vielzahl räumlich verteilter Einzelkomponenten bestehen.

Acknowledgement

I would like to deeply thank my Prof. Gerhard Fettweis for the opportunity to work with him, his kind encouragement, patient guidance, and financial support throughout the last years. Furthermore, I would like to express my deep gratitude to Prof. Frank Jülicher and Dr. Wolfgang Rave for the vivid discussions, inspiring suggestions, criticism, and great effort guiding me throughout my research work. The collaboration with Prof. Frank Jülicher's group at MPI-PKS, especially the work with Dr. Lucas Wetzel and Dr. David J. Jörg who made me enjoy theoretical physics and the different kinds of the Kuramoto models. Many results presented in this thesis arose out of this collaboration. It was a great pleasure and many good memories belong to the time we spend together.

I also want to thank many of the current and former members of the chair for the great atmosphere and the exhilarant chats e.g. during coffee breaks, in particular (in alphabetical order omitting titles) Björn Almeroth, Jens Bartels, Hermann Hensel, Tobias Kadur, Walter Nitzold, Xiaohang Song, Ines Riedel, and Albrecht Wolf.

I wish to thank my family, my parents Christiane and Nikos Pollakis for their warm and constant support, as well as my brother Emmanuel. Last but not least, I would like to express my sincere love and thankfulness to my wife Jitka.

Many thanks to Prof. Gerd Ascheid for accepting to review this thesis.

This work is partly supported by the German Research Foundation (DFG) within the Cluster of Excellence 'Center for Advancing Electronics Dresden', which is gratefully acknowledged.

Contents

‘Δεν ελπίζω τίποτα.
Δε φοβούμαι τίποτα.
Είμαι λέφτερος.’

Νικος Καζαντζακης (1883-1957)

Chapter 1
Introduction

The greek word *synchronous* is composed of the words *sýn* (together) and *chrónos* (time). It describes that events occurs or recurs exactly at the same point in time. For example, during the well-known situation where an audience is clapping for applause, we experience the phenomenon of synchronicity. The audience does not clap by default synchronously with the same rhythm, it undergoes a process termed *synchronisation*.

Probably as one of the first, the famous Dutch researcher Christiaan Huygens (1629-1695) observed and described the phenomenon of synchronisation in a technical sense. He discovered that two pendulum clocks hanging from a common support are ticking synchronously (illustrated in Figure 1.1). His observation is reported by a letter to the Royal Society, and it is referred to as "an odd kind of sympathy" in the Society's minutes [1, p. 19]. Thus, in addition to his precise description he already gave an intuition of the effect's mechanism, the 'sympathy'. Today, this kind of synchronisation is denoted as mutual synchronisation, where synchrony is achieved through mutual interaction.

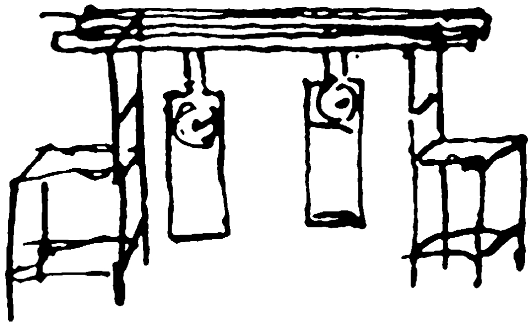

Figure 1.1.
Illustration of two pendulum clocks hanging from a common support, drawn by Christiaan Huygens (image © public domain).

1.1. Synchronisation in modern communication systems

In communication systems, multiple components of a system have to perform their operations in synchrony to ensure functionality. This calls for a coordination among the components that can be provided via a common clock/time reference. Such a common clock reference is important, e.g., for multi-core and multi-processor architectures, systems-on-chip, and antenna arrays [2–10]. All these applications rely on synchronous operation of multiple components, hence, require synchronisation.

1.1.1. Large-scale antenna arrays as a motivating example

As an example, we consider large-scale antenna arrays which are the basis of each massive MIMO system. Massive MIMO is an auspicious candidate for future mobile communication systems and lively discussed in the preparation of 5G. The name is an acronym for a multiple-input multiple output (MIMO) system with a (massively) large number of antennas involved. In conventional MIMO systems the spatial diversity of multi-path propagation is exploited to gain performance in terms of data rate, reliability, energy efficiency, and interference handling [11]. This concept has found its way into mobile communication standards such as Long-Term Evolution [12] and IEEE 802.11n [13], better known by their acronyms LTE and WiFi. As the benefits of MIMO systems rely on the plurality of employed antennas, they scale with their number. However, the number of antennas in commercial LTE base stations is still relatively low.

To exploit the benefits of conventional MIMO systems on a much larger scale, the number of antennas can be increased by several orders of magnitude. These large-scale MIMO systems are subject to an emerging and active research field. As to common knowledge, it is neither clear how to build such large communication systems, nor how to efficiently transmit data through such a large number of antennas exploiting the existing potential [11, 14–17]. One major challenge is to answer the question how to provide phase coherent carrier signals at each antenna element of a large-scale antenna system. These signals are, for example, required at the frequency mixer to shift the baseband signal to higher (radiation) frequencies. The high-frequency carrier signal is usually generated locally at each antenna unit, based on a globally synchronous clock signal which needs to be provided. Due to the high clock frequencies involved and the spatially extended antenna units, precise and robust clock distribution/synchronisation emerges as a critical factor in massive MIMO systems [18, 19].

If we consider massive MIMO systems in combination with millimetre-wave communications, this challenge of synchronisation among the individual antenna units becomes a puzzle. The requirements of synchronisation in millimetre-wave communication systems are stringent and can probably be not fulfilled with available hardware and conventional clock distribution concepts [20]. Hence, the synchronisation can become a limiting factor for large-scale antenna systems especially the ones operating at such high frequencies.

1.1.2. Conventional clock distribution concepts

In the previous sections we have argued that providing a common clock reference is the key to enable high performance and high reliability for state-of-the-art and future communication systems. Many functionalities are based on the requirement that multiple components of the system have to work together in a time coordinated way, e.g. synchrony.

The conventional approach to enable synchronisation of spatially separated entities is to distribute a common clock reference from a *single* master clock all over the system. This concept should be familiar to most of us from

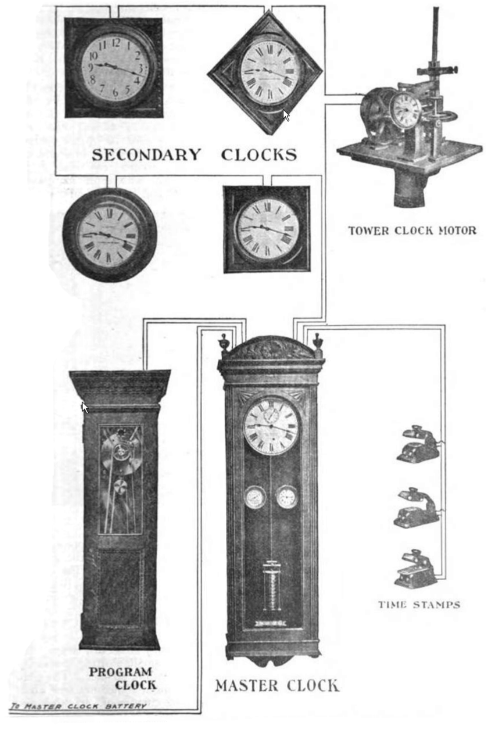

Figure 1.2.
Master clock system used around 1900 as electric time distribution system to keep clocks in sync e.g., in factories, schools, institutions, towers and so on. Diagram by Edwin J. Houston [21, p. 20] (image © public domain).

other subjects than electrical engineering, e.g. from orchestral music. The conductor provides his orchestra with a master/central beat to keep all individual musicians in sync (according to his rhythm). In general, one single master clock dictates the clocking of all connected slave clocks, which follow the master's rhythm. This centralised hierarchical concept is referred to as a master-slave clock network. In the case when multiple hierarchical levels are applied it is referred to as a clock tree, for example including branches, sub-branches, and/or chains of slave clocks.

An early application of an electrical master-slave clock network from around 1900 is illustrated in Figure 1.2. The distribution of a time signal to keep clocks in sync is described as follows:

> *Punctuality is essential to discipline, system and economy in all departments of life. To aid people to be punctual electricity is*

> *used in scores of ways, but even the people in whom these various systems are designed to stimulate promptness do not understand their mysterious working.*
>
> *A modern electric time system consists essentially of a perfectly made, electric, self-winding master clock, which performs the dual function not only of keeping very precise time itself, but of controlling electric impulses to various other time keeping devises of which the system is composed.*
>
> *The illustration shown herewith represents, diagrammatically, a master clock from which radiate electric circuits controlling time stamps; secondary clocks, which keep the exact time of the master clock, a tower clock motor for operating the clocks in a tower or steeple, and a program clock, which is sometimes used in schools. Electrically operated whistles may be added to the system and many other applications will suggest themselves.* (Young [21, p. 20])

We infer from this early description the main properties of electrical master-slave networks. The master clock functions as an autonomous clock of very high precision, and the slave clocks as clocks that are able to adjust their clocking according to the master and keep the exact time. These characteristics are a hallmark of electrical master-slave clock networks and can be retrieved also in applications of modern communication systems.

One aspect of the master-slave clock network in Figure 1.2 is not mentioned in the description quoted: the connection or wiring between master and slave clocks. Whereas signal attenuation effects the synchrony only marginally and can be neglected, the signal propagation and the resulting transmission delays, however, can influence and disturb the synchrony of the system. For the depicted master clock system with a clock frequency of 1 Hz, the wavelength is approx. 30 000 km (considering signal propagation with speed of light). Remembering the circumference of the earth, which is approx. 40 074 km at the equator, the distances involved in the clock network were relatively small compared to the wavelength. The delays introduced between master and slave clocks, respectively the delay differences in between the slaves, are of order of magnitudes smaller than the period of clock and were not relevant for the synchrony of the system.

1.1.3. Is it time to rethink?

Since the early beginning of electrical clock distribution systems many requirements have changed. Modern communication systems operate at much higher clock frequencies, e.g., of up to hundreds of GHz. Due to finite signal propagation velocities, transmission delays occur within clock distribution networks which can be in the order of the oscillation period of the individual clocking signal. In Section 1.1.1 we have argued for the example of massive MIMO systems that synchronisation plays a crucial role in terms of system functionality and performance. In early prototype systems implementing the concept of massive MIMO strenuous efforts are put in providing a common clock reference that satisfies the requirements. Through transmission cables with matched lengths and low clock skew buffering circuits the skew between the single antenna units is tried to keep on a low level [18, 19]. Note that this skew relates to a phase error which is solely introduced by the clock distribution network.

As we can see from the massive MIMO example, it is a challenge to provide a common clock reference in state-of-the-art communication systems. Clock distribution or synchronisation is especially difficult in large extended systems, such as multi-core and multi-processor architectures, system-on-chip, and antenna arrays [2, 3, 6–10]. For such systems, if synchrony is required, a master-slave clock network is commonly implemented to distribute a reference clock signal. Because the reference clock signal is required to be of high precision (as a consequence of the master-slave clock network), it is usually provided externally by a costly module. We now raise the question if this concept provides an appropriate solution for time reference networks in upcoming communication systems.

Following the evolution of communication systems over the last decade, we deduce the following major trends: The number of entities within systems is continuously increasing, and clock frequencies are increasing as well. Hence, future mobile communication systems will consist of a large number of components, that will be by several orders of magnitude larger then today, and will operate at much higher frequencies. Under these circumstances, it is not clear whether a conventional clock distribution concept, following the approach of a master-slave network, can fulfil the synchrony requirements of upcoming mobile communication systems. The well-established master-slave clock tree, implemented across-the-board, has the following conceptional disadvantages which hinder and speak against an implementation in future applications.

(i) Clock trees of multiple hierarchical levels are required to distribute a common reference clock among a large number of entities. However, each hierarchical level increases the synchronisation error [22, 23].

(ii) The robustness of the system decreases linearly with increasing system size [23]. The unidirectional coupling topology, e.g. clock tree and star, does not provide any feed-back to gain robustness.

(iii) Failures or errors of the master clock are not countered or corrected, and propagate through the entire network.

(iv) Perturbations within the distribution network, e.g. noise and cross talk at wires or clock tree forks, are not per se detected and will propagate.

(v) Perfectly equal distances between the master clock and each slave clock are required to avoid time skews at the slaves due to signal propagation, because these directly disturb the synchrony of the slaves.

As a consequence, we seek for a different synchronisation concept that meets the requirements.

1.2. Self-organised clock synchronisation networks

The topic of synchronisation is not restricted to the field of electrical engineering. We have observed different synchronisation concepts in various subjects, e.g. pendulum clocks in the field of mechanics and the mastering conductor in orchestral music. Many more intriguing examples of synchronisation phenomena can be found in nature, specifically in biological systems, where synchronisation is achieved at much larger scale. Especially synchronisation which occurs in a self-organised way has attracted a wide range of experimental and theoretical research interest. Vivid examples are clusters of neurones, coupled genetic oscillators, cardiac pacemaker cells, and flashing fireflies. Common basis of the synchronisation in all these systems are entities that individually show an oscillatory behaviour, e.g. recurring flashing of an individual firefly or cellular genetic oscillators [24–28]. Through mutually coupling of their oscillatory dynamics, these systems are able to synchronise robustly in highly noisy environments. Note that synchrony is achieved solely via interaction of the oscillators and in the *absence* of an entraining master clock.

1.2.1. Vertebrate segmentation clock in zebrafish development

In embryonic development of vertebrates, synchronisation plays a key role. Recent work has provided evidence of how synchronisation gives rise to the segmentation of vertebrae, termed *somitogenesis*[1]. As an example we sketch the tissue segmentation in zebrafish[2] which provides a structure/pattern to the vertebra development. During this process, the segments that later become vertebrae are sequentially formed along the continually elongating body axis in a well-defined order (see Figure 1.3). The sequential segmentation of the so-called somites[3] is observed as a rhythmic process driven by an underlying clock, as proposed by Cooke and Zeeman [30]. This clock is based on a large ensemble of synchronised cellular oscillators, hence, synchronisation via local interaction plays a major role in somitogenesis of zebrafish. Nevertheless, the sequential formation and patterning bears on further steps which we sketch in the following. The *clock-and-wavefront* model [30] gives a description of how patterning can be achieved within two steps/parts:

(i) a clock on tissue scale located at the tail end of the body axis, which is composed of many synchronised cellular oscillators, and

(ii) a repeating wavefront radiating from the tail end, travelling through the tissue along the body axis towards the head end.

The wavefront, characterised by the phase of the cellular oscillators, "freezes" at the arrest front. This front is slowly moving towards the tail end of the body axis, thus, against the travelling wavefronts. This interplay between travelling wavefronts and the moving arrest front forms a phase pattern of regular stripes which in turn triggers an segmentation process of the somites [26]. The given description of the somitogenesis based on a so-called 'segmentation clock' is well supported by experiments [26, 31–34]. Various modelling

[1] Somitogenesis describes the process in which somites form (see [29] for more details).

[2] The zebrafish (*Danio rerio*) is a tropical freshwater fish widely used as model organism for vertebrates.

[3] Somites are structured cell clusters, which are the precursors of the segmented vertebral column (see [29] for more details).

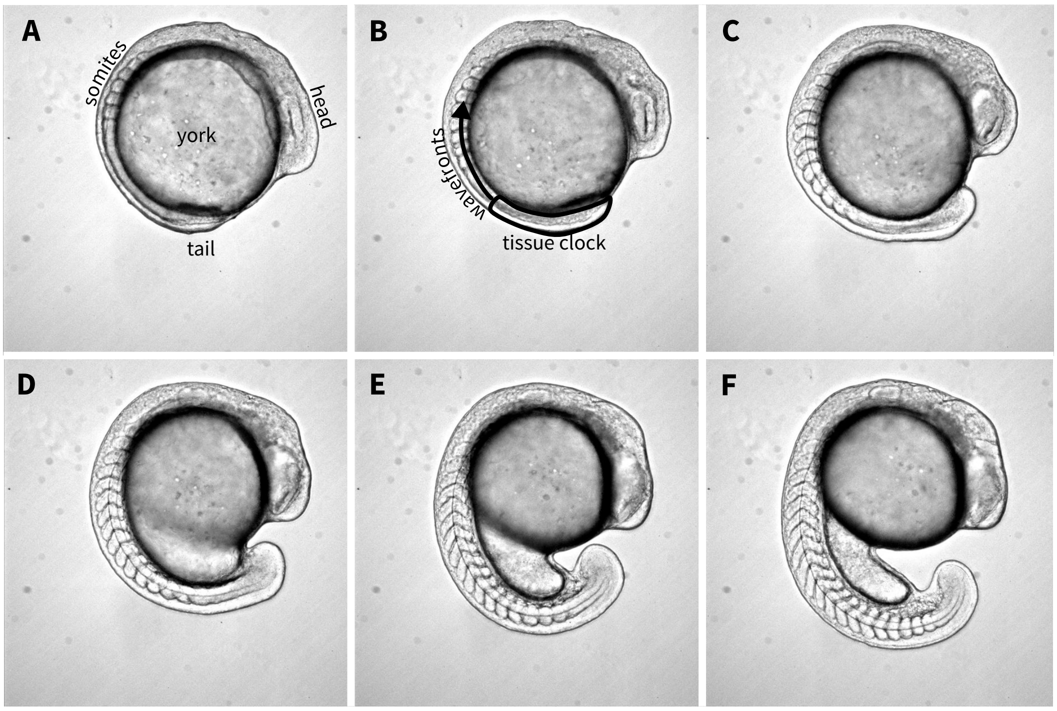

Figure 1.3.
Segmentation of the embryonic body axis of the zebrafish (*Danio rerio*) embryo. Lateral view of the zebrafish embryo under brightfield microscopy. Snapshots at different stages of segmentation: 5 somites (**A**), 8 somites (**B**), 11 somites (**C**), 14 somites (**D**), 17 somites (**E**), and 20 somites (**F**) (images courtesy of Daniele Soroldoni).

approaches have been made to theoretically describe segmentation clocks in many organisms [35–39]. Figure 1.4 shows schematically and simplified the somitogenesis in zebrafish based on the segmentation clock theory.

As proposed in the clock-and-wavefront model, the tissue clock is based on an ensemble of cellular oscillators that synchronously perform periodic expressions of so-called cyclic genes. These kinds of cellular oscillators have been extensively studied experimentally [40–51] and theoretically [52–56]. To provide a clock on tissue scale, the cellular oscillators have to synchronise the cycles of recurring gene expression, that are described by their phases. Intercellular coupling is the key for synchronisation through which the oscillators are able to influence the phases of adjacent oscillators. The complex signalling between cells is implemented via macromolecules, including synthesis and trafficking [57, 58], similar to transmitter and receiver in communication systems. Because these processes are not instantaneous, the coupling

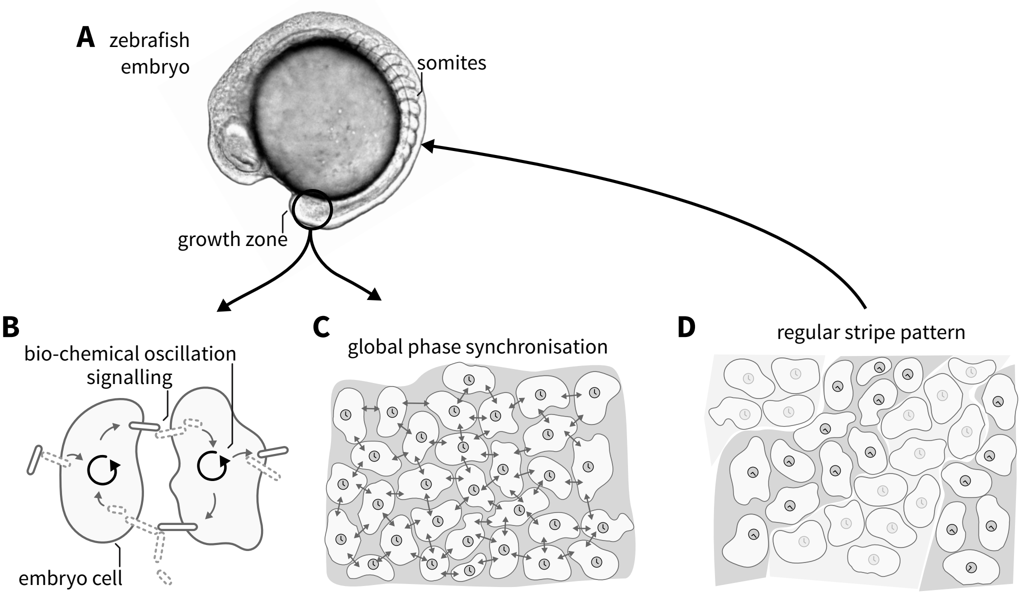

Figure 1.4.
Segmentation of the embryonic body axis of the zebrafish (*Danio rerio*) embryo. (**A**) Lateral view of the zebrafish embryo under brightfield microscopy (images courtesy of Daniele Soroldoni). (**B**) Schematic illustration of the bio-chemical oscillators at cellular level and the Delta-Notch signalling pathway. (**C**) Self-organised synchronisation of spatially distributed cellular oscillators via mutually coupling with delay. (**D**) Somite segmentation based on self-organised synchronisation at tissue level.

is subject to time delays [35, 57]. For zebrafish these delays can be in the order of the oscillation period of the cellular oscillators, and influence the synchronisation of the ensemble [59–62]. Despite the coupling delay, these networks of oscillators tend to synchronise. Hence, the tissue clock emerges from the phase dynamics of the oscillator ensemble.
Note that synchronisation is achieved solely via local interaction in a network consisting of a large number of spatially distributed cellular oscillators. Neither a predominant oscillator entraining others, nor a hierarchical structure is involved. Furthermore, the achieved synchronous state is robust against perturbations, i.e. high noise levels that are present in biological systems [58, 63, 64].

1.2.2. Bio-inspired synchronisation in communication systems

Biological systems can be a rich source of inspiration to advance communication systems [65]. Regarding the challenge of providing a common reference clock in large and spatially extended systems, e.g. massive MIMO systems, synchronisation concepts in biological systems can serve as a model. Robust synchronisation is achieved through local interaction between oscillators, in the *absence* of an entraining master.

Abstracting from the complexity of biological systems is the key to gain a general understanding of the underlying synchronisation mechanisms. First theoretical contributions for biological system are attributable to Winfree [66]. Following his ideas of how to model rhythmic processes in nature including a large set of interacting oscillators, the *Kuramoto model* was developed [67]. Based on the seminal work of Kuramoto many extensions have been investigated in various fields of research, e.g. considering inhomogeneous oscillators and/or delayed coupling [59–62, 68–73].

Synchronisation phenomena in biological systems also attracted interest of communication engineers, including the concept of self-organised synchronisation in networks of mutually coupled oscillators. The topic first occurred under the term ‘organic synchronisation’ mainly introduced by V. E. Beneš and his colleagues at Bell Telephone Laboratories [74–77]. During the last decades, synchronisation in oscillator networks enabled through mutual coupling has been considered in many applications, for example in network synchronisation, multiprocessor systems, system-on-chip, antenna arrays, and analog and digital circuitry [23, 78–88]. Even though, other synchronisation concepts are prevailingly applied in all these communication systems.

1.2.3. Rethinking clock distribution for future communication systems

The bio-inspired concept of self-organised synchronisation has been known to electrical engineers for a long time [23]. But up to our knowledge, it has not been applied commercially in communication systems for many different reasons. Nevertheless, the clock distribution and synchronisation in modern and future communication system faces major challenges as pointed out in Section 1.1.3. Extrapolating the trends of continuously increasing number of entities/components and increasing clock frequencies, future communication systems will have properties, e.g. system size and (relative) transmission delays, which we see in biological systems.
As an example, we compare the clock synchronisation required in a large-scale antenna array (e.g. for massive MIMO) with the tissue clock observed during embryonic development in zebrafish. Both systems consist of hundreds, up to thousands, of entities (electrical or cellular oscillators, antenna units) and the functionality of the systems relies on their synchrony. Without synchrony the tissue clock would not emerge during somitogenesis in zebrafish development, and the potential performance gains in massive MIMO, e.g. based on multi-path propagation and beam steering, would diminish. Another similarity is that in communication systems as well as in biological systems the signalling between coupled oscillators is delayed in the order of the oscillation period, due to the relation between spatial distances and signal propagation velocities. Even more similarities can be found, e.g. the expected precision of the deployed oscillators. In large antenna arrays it is expected that low-cost low-precision components will be used to keep the hardware cost to a minimum [15]. The tissue clock observed during embryonic development in zebrafish consists of cellular oscillators, which are due to internal fluctuations per se of low precision [39]. However, it has been shown that coupling between the oscillators can increase precision of the tissue clock by more than one order of magnitude [39].

1.3. Content of this thesis

In this thesis, we propose a concept of self-organised synchronisation, based on a network of mutually delay-coupled electronic oscillators. Through coupling the oscillators are able to synchronise their clock signals such that network-wide synchronisation can be achieved. The spatial distances and finite signal propagation velocities induce transmission delays in the coupling between the oscillators. Intuitively, these delays seem to inhibit the process of synchronisation. This raises the question of how self-organised synchronisation can be achieved in such networks of electronic oscillators in the presence of transmission delays.

In Chapter 2, we introduce the single oscillator as the basic clock entity of the network and develop a phase description. We extend this description to a phase model of a network of mutually coupled oscillators with delayed transmission. Here, we use a deterministic phase model (no stochasticity or noise) and investigate the basic network behaviour in an ideal environment.

In Chapter 3, we give a condensed overview on the state of the art in understanding the dynamics of coupled oscillators, especially the onset of synchronisation. Furthermore, we discuss the relation of our phase model to the class of Kuramoto models and point out its novelty and uniqueness.

In Chapter 4, we investigate the existence of frequency-synchronised states with constant phase differences between the oscillators, e.g. in-phase synchronised states with zero phase difference. For the introduced synchronised states we determine the stability via an analysis of the decay or growth of perturbations close to the synchronised states. Moreover, we give numerical examples focussing on the in-phase synchronised state which is characterised by zero phase difference between all oscillators.

In Chapter 5, we show experimental measurement results for small networks of mutually delay-coupled oscillators. We show results for the existence of the previously introduced synchronised states and compare the predicted synchronisation dynamics of our phase model to a real system.

In Chapter 6, we give an overview on topics we only briefly touched but strongly suggest for future work. This selection of topics includes networks of inhomogeneous oscillators, robustness against noise, and booting strategies.

We conclude the thesis in Chapter 7, where we summarise our work.