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**Zhijun Rong**

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**Performance Evaluation of Coordinated Multi-Point  
(CoMP) Uplink under Consideration of Realistic  
Channel Estimation**

Dipl.-Ing.

**Zhijun Rong**

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# Abstract

Wireless and mobile communications become more popular in the recent years. With the introduction of new applications in the consumer market, such as mobile internet services on smart phones, mobile video and gaming, the demands for higher data rate at a reasonable cost per bit have increased a lot. Since the frequency band for mobile communications is a rare resource and therefore very expensive, a frequency-reuse factor of 1 is often used in the current cellular systems. This causes a disturbing effect termed as "inter-cell interference", which has to be combated to improve spectral efficiency.

The latest development is specified in the Long Term Evolution-Advanced (LTE-A) standard, where new schemes literally referred to as "Coordinated Multi-Point" (CoMP) are applied to mitigate or even exploit inter-cell interference by processing signals connected to multiple user equipments (UEs) with base station (BS) cooperation. From theory the CoMP schemes are proven very promising to increase spectral efficiency significantly, especially at the cell edge areas. However, there still exist some critical issues for the practical implementation of the CoMP schemes. Beside the synchronization and backhaul optimization problems, a main issue is the pilot and channel estimator design to ensure an accurate and reliable estimation of the channel links so that the performance of the CoMP schemes are not degraded too much.

This thesis focuses on the problem of channel estimation of the CoMP *uplink* system. Two specific pilot and estimator designs are proposed and illustrated with a running example. One applies frequency-orthogonal (FO) pilots, where non-zero pilot sequences of each UE are allocated to equally-spaced disjoint subcarriers. This design avoids multi-user interference completely but requires a two-step estimation procedure, where we first estimate channel coefficients at the pilot subcarriers and then interpolate coefficients between these pilot subcarriers. The other applies code-orthogonal (CO) pilots. We transmit pilot sequences of all UEs at the allocated pilot subcarriers simultaneously and this causes multi-user interference. A major contribution of this work is that we evaluate the leakage effect caused by this pilot allocation scheme and derive a simple rule of the optimal shift parameter in the pilot design. The numerical results verify that this design mitigates multi-user interference significantly. Moreover, we propose a new CO pilot design using Frank-Zadoff-Chu (FZC) sequences. Compared with other pilot designs, such as the conventional LTE pilot sequences and phase shift (PS) sequences, it turns out to be the best option. Correspondingly, a simplified channel estimation with model mismatch is proposed which is practical to be implemented in the real CoMP uplink system.

Beside the specific pilot and estimator design, we also verify the equivalence of the channel estimators using frequency-orthogonal and code-orthogonal pilots analytically. Follow-

ing the established theory in the literature, we characterize the performance of channel estimation by the mean squared error (MSE) of the channel estimation error, consisting of two parts, which are the interpolation error part and noise error part, respectively. Under the comparable system parameters, we can not draw a unified conclusion which pilot design is better by a direct MSE comparison. Next, the system performance degradation due to channel estimation error is characterized by a new metric of the SNR degradation before detection. Numerical results verify that the channel estimator using code-orthogonal pilots causes less degradation than that using frequency-orthogonal pilots. Finally, we review the specific parameter design of code-orthogonal pilots and estimator with the focus on the optimization of the pilot boost parameter under the total data/pilot power constraint. A realistic sub-optimum of pilot boost is derived and the numerical results show that it achieves the system performance degradation very close to that with the optimal pilot boost.

Another main contribution from this work is that we propose a new design of the unbiased minimum mean square error (MMSE) filter for joint data detection in the CoMP uplink system. By properly characterizing channel estimation error in the filter design, we achieve the satisfactory bit error rate (BER) performance. Here, a realistic filter design without knowledge of the true channel statistics is also derived and numerical results show that the performance loss is negligible for all considered channel models.

In summary, it is shown in this work that with the proper pilot and channel estimator design, together with the proper design of the joint data detector under consideration of channel estimation error, we can make channel estimation a solvable problem in the CoMP uplink system.

# Zusammenfassung

Die drahtlose mobile Kommunikation ist immer populärer geworden in den letzten Jahren. Mit der Einführung von neuen Anwendungen im Markt, wie zum Beispiel mobile Internet-Dienste auf Smartphones, mobilen Video und Spiel, steigt die Anforderung für höhere Datenrate zu einem vernünftigen Kosten pro Bit immens. Da das Frequenzband für Mobilfunk eine rare Ressource und daher sehr teuer ist, wird immer mehr eine Frequenz-Wiederverwendung Faktor von 1 an den jetztigen zellulären Systemen verwendet. Dies führt zu einem störenden Effekt, bezeichnet als "Interzelleninterferenz", was zu bekämpfen gilt, um die spektrale Effizienz zu steigern.

Die neueste Entwicklung ist spezifiziert in dem Long Term Evolution Advanced (LTE-A) Standard, wo neue Schemen angewendet werden, was als "Coordinated Multi-Point" (CoMP) bezeichnet sind, um die Interzelleninterferenz zu mildern oder sogar auszunutzen, durch Verarbeitung von Signalen von mehreren User Equipments (UEs) mit der Basisstation (BS) Kooperation. Von der Theorie is bekannt dass die CoMP-Systeme viel versprechend sind, die spektrale Effizienz deutlich zu erhöhen, vor allem auf die Randbereichen von den Zellen. Allerdings gibt es immer noch einige kritische Punkten für die praktische Umsetzung der CoMP Systemen. Neben den Synchronisation und Backhaul Optimierungproblemen ist der Pilot und Kanalschätzer Design ein Hauptproblem, um eine genaue und zuverlässige Schätzung der Kanäle sicherzustellen, so dass die Leistung der CoMP-Systemen nicht zu viel degradiert wird.

Diese Arbeit konzentriert sich auf das Problem der Kanalschätzung der CoMP *Uplink* Systemen. Zwei spezifische Pilot und Kanalschätzer Designs sind vorgeschlagen und veranschaulicht mit einem Beispiel. Der einer verwendet Frequenz-orthogonal (FO) Piloten, wo Nicht-Null-Pilot-Sequenzen von jedem UE den disjunkten Unterträgern mit gleichem Abstand zugeordnet sind. Dieser Design vermeidet Multi-User-Interferenz vollständig, aber erfordert auch ein zweistufige Schätzverfahren, wo wir erst die Kanalkoeffizienten bei den Pilotunterträgern schätzen und dann die Koeffizienten zwischen diesen Pilotunterträgern interpolieren müssen. Der andere verwendet Code-orthogonal (CO) Piloten. Wir übertragen Pilotsequenzen aller UEs an den zugeordneten Pilotunterträger gleichzeitig und das verursacht Multi-User-Interferenz. Ein wesentlicher Beitrag dieser Arbeit liegt darin dass wir den Leakage-Effekt analysieren, die durch diesem Pilotschema verursacht ist und daraus eine einfache Regel für das optimale Shift-Parameter im Pilot Design einleiten. Die numerischen Ergebnisse bestätigen, dass dieser Design Multi-User-Interferenz deutlich mildert. Darüber hinaus schlagen wir einen neuen CO Pilot-Design mit Frank-Zadoff-Chu (FZC) Sequenzen vor. Gegenüber anderen Pilot Designs, wie zum Beispiel die herkömmlichen LTE Pilotsequenzen und Phase Shift (PS)-Sequenzen, stellt sich heraus dass unsere neuen Pi-

lotsequenzen die beste Option sind. Entsprechend wird auch ein vereinfachtes Verfahren zur Kanalschätzung mit Mismatch vorgeschlagen, die in dem realen CoMP Uplink-System umgesetzt werden kann.

Neben den spezifischen Pilot und Schätzer Designs haben wir auch die Äquivalenz der Kanalschätzern mit Frequenz-orthogonal und Code-orthogonal Piloten analytisch verifiziert. Basierend auf die etablierten Theorie in der Literatur charakterisieren wir die Leistung der Kanalschätzung mit dem mittleren quadratischen Fehler (MSE) der Kanalschätzung Fehler, die aus zwei Teilen bestehen, einer dem Interpolationsfehler und der andere Rauschenfehler entspricht. Unter den vergleichbaren System-Parametern, können wir eine einheitliche Schlussfolgerung nicht ziehen, welcher Pilot-Design besser ist durch einen direkten Vergleich von MSE Werten. Als nächstes wird die Systemleistung Degradation durch Kanalschätzung Fehler gekennzeichnet durch einem neuen Metrik, die SNR-Degradation vor der Detektion. Numerische Ergebnisse bestätigen, dass der Kanalschätzer mit Verwendung von Code-orthogonal Piloten weniger Degradation verursacht als den mit Verwendung von Frequenz-orthogonal Piloten. Schließlich haben wir die spezifischen Parameter Designs für Code-orthogonale Pilot und Schätzer diskutiert, mit dem Fokus auf die Optimierung des Pilot-Boost-Parameter unter der gesamten Data/Pilot Leistungseinschränkung. Ein realistische suboptimale Wert von Pilot-Boost wird abgeleitet und die numerischen Ergebnisse zeigen, dass es das System Degradation Performance erreichen kann, das sehr nahe zu dem mit dem optimalen Pilot Boost zu erreichen ist.

Ein weiterer Beitrag dieser Arbeit besteht darin, dass wir einen neuen Design des unbiased minimalen mittleren quadratischen Fehlers (MMSE)-Filter für Joint Data Detektion in dem COMP-Uplink-System vorgeschlagen haben. Durch den angepassten Kanalschätzfehler in dem Filter-Design, können wir eine zufriedenstellende Bitfehlerrate (BER) erreichen. Hier wird auch ein realistischer Filter-Design ohne Kenntnis von der realen Kanalstatistiken abgeleitet und numerische Ergebnisse zeigen, dass der Performance-Verlust vernachlässigbar ist für alle betrachteten Kanalmodellen.

Zusammenfassend ist es in dieser Arbeit gezeigt, dass wir die Kanalschätzung in dem CoMP-Uplink System als ein lösbares Problem betrachten können, wenn wir die Pilot-Sequenzen und Kanalschätzer richtig entwerfen und dann den Joint Data Detektor unter Berücksichtigung der Kanalschätzung Fehler anpassen.



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# Abbreviations

2D	Two-Dimensional
3GPP	3rd Generation Partnership Project
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BMBF	German Federal Ministry of Education and Research
BS	Base Station
CIR	Channel Impulse Response
CO	Code-Orthogonal
CoMP	Coordinated Multi-Point
CP	Cyclic Prefix
CTF	Channel Transfer Function
CU	Central Unit
DFT	Discrete Fourier Transform
EASY-C	Enablers of Ambient Services and Systems: Part C
FFT	Fast Fourier Transform
FO	Frequency-Orthogonal
FZC	Frank-Zadoff-Chu
HSPA	High Speed Packet Access
IDFT	Inverse Discrete Fourier Transform
LIE	Linear Interpolation and Extrapolation
LTE	Long Term Evolution
LTE-A	Long Term Evolution Advanced
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MSE	Mean Squared Error
OFDM	Orthogonal Frequency Division Multiplexing
p.c.e.	Perfect Channel Estimation
PCE	Perfect Channel Estimation
PDP	Power Delay Profile

---

Ped A	Modified Pedestrian A Channel Model
PS	Phase Shift
PUCCH	Physical Uplink Control Channel
PUSCH	Physical Uplink Shared Channel
QAM	Quadrature Amplitude Modulation
RS	Reference Signal
RU	Resource Unit
SC-FDMA	Single-Carrier Frequency Division Multiple Access
SIR	Signal to Interference Ratio
SNR	Signal to Noise Ratio
UE	User Equipment
Veh A	Vehicular A Channel Model
WF	Wiener Filtering
WSSUS	Wide-Sense Stationary Uncorrelated Scattering
ZF	Zero-Forcing

# List of Symbols

## Operations and Functions

$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian(conjugate transpose)
$(\cdot)^*$	Complex conjugate
$(\cdot)^{-1}$	Inverse of a matrix
$(\cdot)^+$	Pseudo-inverse of a matrix
$p(\cdot)$	Probability density von $(\cdot)$
$\Pr(\cdot)$	Probability of $(\cdot)$
$\mathcal{E}(\cdot)$	Expected value of $(\cdot)$
$(\cdot)_N$	the modulo $N$ operation
$\exp(\cdot)$	Exponential of a number
$ \cdot $	Absolute value of a scalar, determinant of a matrix or cardinality of a set
$\ \cdot\ _2$	the Euclidean norm of the enclosed vector
$\ \cdot\ _F$	the Frobenius norm of the enclosed matrix
$(\hat{\cdot})$	estimates of an unknown value
$(\tilde{\cdot})$	hypothesis of an unknown value
$\text{tr}(\cdot)$	Trace of a matrix
$\text{rank}(\cdot)$	Rank of a matrix
$\text{diag}(\cdot)$	Diagonal matrix with the elements of the enclosed vector along its main diagonal
$\text{blockdiag}(\cdot)$	a block diagonal concatenation of matrix input argument
$\text{dg}(\cdot)$	Forms a diagonal matrix from the leading diagonals of a matrix
$\arg(\cdot)$	Argument(phase) of a complex quantity
$\text{gcd}(\cdot)$	Greatest common divisor
$\text{Re}(\cdot)$	Real part of a complex value
$\text{Im}(\cdot)$	Imaginary part of a complex value
$\text{primes}(\cdot)$	All the prime number smaller than the given
$\in$	Element of
$\triangleq$	definition

$\forall$	For all
$\subseteq$	Subset
$\sum$	Sum
$\prod$	Product
$\frac{\partial}{\partial t}$	Derivative
$*$	Convolution
$\otimes$	Kronecker product
$\circledast$	Cyclic convolution
$\odot$	Element by element multiplication
$\mathbb{C}^{n \times m}$	Set of complex numbers of dimension $n \times m$
$\mathbb{R}^{n \times m}$	Set of real numbers of dimension $n \times m$
$\mathbb{Z}$	Set of integers
$\Pi$	Interleaver
$\Pi^{-1}$	Inverse Interleaver
$\max_{\mathbf{x}}\{\cdot\}$	Maximum of $\{\cdot\}$ over $\mathbf{x}$
$\min_{\mathbf{x}}\{\cdot\}$	Minimum of $\{\cdot\}$ over $\mathbf{x}$
$\lceil \cdot \rceil$	closest upper integer of a ratio
$\lfloor \cdot \rfloor$	closest lower integer of a ratio

## Symbols

$a_{u_p}^{(\nu')}$	the $\nu'$ -th root FZC sequence of length $N_p$
$a_{u_p}^{(\nu', \mu)}$	cyclically shift version of the $\nu'$ -th root FZC sequence $a_{u_p}^{(\nu')}$
$a_{u_p}^{(\nu', \mu)}$	cyclic extension of $b_{u_p}^{(\nu', \mu)}$ to length $N_p$
$b_{u_p}^{(\nu')}$	the $\nu'$ -th root FZC sequence of length $N_g$
$b_{u_p}^{(\nu', \mu)}$	cyclically shift version of the $\nu'$ -th root FZC sequence $b_{u_p}^{(\nu')}$
$c_{u_p}^{(\nu')}$	the $\nu'$ -th root FZC sequence of length $N_{\text{ff}}$
$c_{u_p}^{(\nu', \mu)}$	cyclically shift version of the $\nu'$ -th root FZC sequence $c_{u_p}^{(\nu')}$
$B_c$	channel coherence bandwidth
$BER_{\text{avg}}$	average bit error rate over all UEs
$C$	speed of light
$C^\mu$	subcarrier index of the first pilot signal of UE $\mu$
$d^{(\nu, \mu)}$	distance between UE $\mu$ and BS $\nu$
$D_f$	pilot spacing in frequency
$E_0$	data signal energy of the baseline system
$E_d$	data signal energy
$E_p$	pilot signal energy
$E_{\text{tot}}$	total data/pilot power constraint

$f_c$	carrier frequency
$f_{d,\max}$	maximal Doppler frequency
$f_s$	sampling frequency
$f_{sc}$	subcarrier spacing
$G_{u_d,fo}^{(\nu,\mu)}$	filter gain with FO pilots
$G_{u_p,co}^{(\nu,\mu)}$	filter gain with CO pilots
$l$	OFDM symbol index
$l_d$	data signal OFDM symbol index
$l_p$	pilot signal OFDM symbol index
$l_p^{(\mu)}$	pilot signal OFDM symbol index of UE $\mu$
$l_{\text{usd}}$	unoccupied pilot signal OFDM symbol index
$L_{\text{ch}}$	maximum CIR length
$i$	channel tap index
$MSE_{\text{fd}}$	MSE in the frequency domain of the channel estimator using CO pilots
$MSE_{\text{fd,sim}}$	average MSE of the simplified channel estimator using CO pilots over $\mathcal{U}_p$ and all UEs
$n$	subcarrier index
$n_d$	data signal subcarrier index
$n_p$	pilot signal subcarrier index
$n_{p,\text{first}}$	the first entry from the set of pilot subcarrier indices $n_p$
$n_{p,\text{last}}$	the last entry from the set of pilot subcarrier indices $n_p$
$n_p^{(\mu)}$	pilot signal subcarrier index of UE $\mu$
$n_{p,\text{LIE}}^{(\mu,n_d)}$	index of pilot subcarrier closest to the data subcarrier index $n_d$
$n_{\text{usd}}$	unoccupied pilot signal subcarrier index
$N_0$	noise variance
$N_{\text{bs}}$	number of BSs
$N_{\text{cp}}$	number of cyclic prefix samples
$N_{\text{cs}}^{(\mu)}$	UE specific shift parameter
$N_d$	number of data signals per UE in a subframe
$N_{\text{fft}}$	FFT size
$N_g$	the largest prime number with $N_g < N_p$
$N_{\text{offset}}$	offset of the position of the first allocated resource unit
$N_p$	number of subcarriers allocated to pilot signals in each OFDM pilot symbol
$N'_p$	number of subcarriers with non-zero pilot sequences per UE
$N_{\text{RU}}$	number of allocated resource units
$N_{\text{ue}}$	number of UEs
$P_b^{(\mu)}$	bit error rate of UE $\mu$
$r_c$	radius of the cooperation cluster

$R_c$	cell radius
$S_p$	pilot boost factor
$S_{p,\text{opt}}$	optimum $S_p$
$S_{p,\text{subopt}}$	sub-optimum $S_p$
$SIR^{(\nu,\mu)}$	signal-to-interference ratio parameter for UE $\mu$ at BS $\nu$
$SIR_{\text{avg}}^{(\nu)}$	average of $SIR^{(\nu,\mu)}$ over all UEs at BS $\nu$
$SNR_{\text{comp,ce},u_d}^{\text{eff}(\nu)}$	effective pre-detection SNR with the centralized CoMP mode with channel estimation
$SNR_{\text{comp,pce},n,l}^{(\nu)}$	instantaneous pre-detection SNR with the centralized CoMP mode under perfect channel estimation
$SNR_{\text{comp,pce},n,l}^{\text{eff}(\nu)}$	effective pre-detection SNR with the centralized CoMP mode under perfect channel estimation
$SNR_{\text{conv,pce},n,l}^{(\nu)}$	instantaneous pre-detection SNR with the conventional mode under perfect channel estimation
$SNR_{\text{conv,pce},n,l}^{\text{eff}(\nu)}$	effective pre-detection SNR with the conventional mode under perfect channel estimation
$SNR_{\text{JD},u_d,\text{UB,CE}}^{(\mu)}$	SNR of the detected data signal $X_{u_d}^{(\mu)}$ at the output of unbiased MMSE filter with channel estimation error
$SNR_{\text{JD},u_d,\text{UB,PCE}}^{(\mu)}$	SNR of the detected data signal $X_{u_d}^{(\mu)}$ at the output of unbiased MMSE filter with perfect channel estimation
$SNR_{\text{JD},u_d,\text{ZF,CE}}^{(\mu)}$	SNR of the detected data signal $X_{u_d}^{(\mu)}$ at the output of ZF filter with channel estimation error
$SNR_{\text{JD},u_d,\text{ZF,PCE}}^{(\mu)}$	SNR of the detected data signal $X_{u_d}^{(\mu)}$ at the output of ZF filter with perfect channel estimation
$SNR_d$	per data subcarrier SNR
$SNR_{p,\text{co}}$	per pilot subcarrier SNR with CO pilots
$SNR_{p,\text{fo}}$	per pilot subcarrier SNR with FO pilots
$SNR_{p,\text{fo,max}}$	largest expected SNR for channel estimation with frequency-orthogonal pilots
$T_c$	coherence time
$T_{\text{cp}}$	cyclic prefix length
$T_s$	sampling duration
$u_d$	data signal count index
$u_p$	pilot signal count index
$\mathbb{U}$	set of indices allocated to data and pilot signals in a subframe
$\mathbb{U}_d$	set of indices allocated to data signals per UE
$\mathbb{U}_p$	set of indices allocated to pilot signals per UE
$\mathbb{U}_p^{(\mu)}$	set of indices with the non-zero pilot signals of UE $\mu$
$\mathbb{U}_{\text{usd}}$	set of indices reserved for pilots but not occupied
$v_{\text{ue}}$	UE speed
$Z_{n_p,l_p}^{(\nu)}$	AWGN at BS $\nu$ at the $(n_p, l_p)$ -th resource element

$\alpha$	ratio of the cooperation cluster w.r.t. $R_c$
$\Delta_{cs}$	UE-independent shift parameter
$\Delta_{cs}^{(\mu, \mu')}$	shift difference between UE $\mu$ and $\mu'$
$\Delta\gamma_{\text{comp,eff}}^{(\nu)}$ (dB)	average of $\Delta\gamma_{\text{comp,eff},u_d}^{(\nu)}$ (dB) over $\mathbb{U}_d$ and all BSs
$\Delta\gamma_{\text{comp,eff}}^{(\nu)}$ (dB)	average of $\Delta\gamma_{\text{comp,eff},u_d}^{(\nu)}$ (dB) over $\mathbb{U}_d$
$\Delta\gamma_{\text{comp,eff},u_d}^{(\nu)}$ (dB)	effective pre-detection SNR degradation in decibels due to channel estimation error
$\Delta\gamma_{\text{comp,eff},u_d}^{(\nu)}$ (linear)	effective pre-detection SNR degradation in scalar due to channel estimation error
$\Delta\gamma_{\text{coop,pce},n,l}^{(\nu)}$ (dB)	effective pre-detection SNR degradation in decibels due to the centralized CoMP mode
$\Delta_n$	subcarrier distance
$\Delta_{\text{opt,cs}}$	optimized $\Delta_{cs}$
$\eta$	path-loss exponent
$\mu$	UE index
$\nu$	BS index
$\nu'$	root number
$\Phi_{H_{u_d}^{(\nu,\mu)} H_{u_d}^{(\nu,\mu)}}$	auto-correlation of scalar $H_{u_d}^{(\nu,\mu)}$
$\Phi_{HH}^{(\nu,\mu)}(\Delta_n)$	one-dimensional correlation function of channel in frequency
$\rho^{(\nu,\mu)}$	path-loss parameter for the receive-transmit antenna pair $(\nu, \mu)$
$\sigma_E^{2(\nu,\mu)}$	average MSE of channel estimation error over the entire set $\mathbb{U}_d$
$\sigma_{E,u_d}^{2(\nu,\mu)}$	MSE of channel estimation error with FO pilots
$\sigma_{E,u_d,\text{WF,opt}}^{2(\nu,\mu)}$	minimum MSE of the optimal Wiener filtering
$\sigma_{E,u_p}^{2(\nu,\mu)}$	MSE of channel estimation error with CO pilots
$\sigma_{E,\text{JD},u_d}^{2(\mu)}$	variance of the detection error of UE $\mu$ at subcarrier $u_d$ for the general joint detection filter
$\sigma_{E,\text{JD},u_d,\text{UB,CE}}^{2(\mu)}$	variance of the detection error of UE $\mu$ at subcarrier $u_d$ with unbiased MMSE filter with channel estimation error
$\sigma_{E,\text{JD},u_d,\text{ZF,CE}}^{2(\mu)}$	variance of the detection error of UE $\mu$ at subcarrier $u_d$ with ZF filter with channel estimation error
$\sigma_{E,\text{JD},u_d,\text{UB,PCE}}^{2(\mu)}$	variance of the detection error of UE $\mu$ at subcarrier $u_d$ with unbiased MMSE filter under perfect channel estimation
$\sigma_{E,\text{JD},u_d,\text{ZF,PCE}}^{2(\mu)}$	variance of the detection error of UE $\mu$ at subcarrier $u_d$ with ZF filter under perfect channel estimation
$\sigma_{\check{H},u_d}^{2(\nu,\mu)}$	variance of $\check{H}_{u_d}^{(\nu,\mu)}$ with FO pilots
$\sigma_{\check{H},u_p}^{2(\nu,\mu)}$	variance of $\check{H}_{u_p}^{(\nu,\mu)}$ with CO pilots
$\sigma_i^2$	power delay profile of the $i$ -th channel tap



$\sigma_{1,u_d}^{2(\nu,\mu)}$	MSE of interpolation error with FO pilots
$\sigma_{1,u_p}^{2(\nu,\mu)}$	MSE of interpolation error with CO pilots
$\sigma_{N,u_d}^{2(\nu,\mu)}$	MSE of noise error with FO pilots
$\sigma_{N,u_p}^{2(\nu,\mu)}$	MSE of noise error with CO pilots
$\sigma_Z^2$	noise variance for channel estimation
$\sigma_{\tilde{Z}}^2$	variance of the normalized noise for channel estimation
$\sigma_{\tilde{Z},JD}^2$	variance of the normalized noise for joint data detection
$\sigma_{\tilde{Z},\min}^2$	minimum expected variance of the normalized noise for channel estimation
$\sigma_{\tilde{Z},\text{mis}}^2$	mismatched variance of the normalized noise for channel estimation
$\sigma_{\tilde{Z},\text{opt}}^2$	true variance of the normalized noise for channel estimation
$\tau_i$	delay of the $i$ -th channel tap
$\tau_{\max}$	maximum delay of CIR

## Vector elements

$E_{u_d}^{(\nu,\mu)}$	channel estimation error at index $u_d$ allocated to data signal for the receive-transmit antenna pair $(\nu, \mu)$
$G_{n,l}^{(\nu,\mu)}$	coefficient from the channel transfer function for the receive-transmit antenna pair $(\nu, \mu)$ without path-loss
$H_{n,l}^{(\nu,\mu)}$	coefficient from the channel transfer function for the receive-transmit antenna pair $(\nu, \mu)$ with path-loss
$H_{u_p}^{(\nu,\mu)}$	channel transfer function at index $u_p^{(\mu)}$ between BS $\nu$ and UE $\mu$
$\check{H}_{u_d}^{(\nu,\mu)}$	signal part of the channel estimate at index $u_d$ allocated to data signal for the receive-transmit antenna pair $(\nu, \mu)$
$\hat{H}_{u_d}^{(\nu,\mu)}$	channel estimate at index $u_d$ allocated to data signal for the receive-transmit antenna pair $(\nu, \mu)$
$I_{u_d}^{(\nu,\mu)}$	interpolation error at index $u_d$ allocated to data signal for the receive-transmit antenna pair $(\nu, \mu)$
$R_{u_p}^{(\nu,\mu)}$	pilot observation of UE $\mu$ at index $u_p^{(\mu)}$ at BS $\nu$
$S_{n_p,l_p}^{(\mu)}$	pilot signal from UE $\mu$ at subcarrier $n_p$ in the $l_p$ -th OFDM symbol
$X_{n_d,l_d}^{(\mu)}$	data signal from UE $\mu$ at subcarrier $n_d$ in the $l_d$ -th OFDM symbol
$Z_{n,l}^{(\nu)}$	AWGN at BS $\nu$ at the $(n, l)$ -th resource element
$Z_{u_p}^{(\nu,\mu)}$	noise term at index $u_p^{(\mu)}$ at BS $\nu$

## Vectors and Matrices in Chapter 2

$\underline{\mathbf{F}}$	$[N_{\text{fft}} \times N_{\text{fft}}]$ : DFT matrix
$\mathbf{g}^{(\nu,\mu)}$	$[L_{\text{ch}} \times 1]$ : CIR for the receive-transmit antenna pair $(\nu, \mu)$ without path-loss
$\underline{\mathbf{G}}_{n,l}$	$[N_{\text{bs}} \times N_{\text{ue}}]$ : MIMO channel matrix without path-loss at the $(n, l)$ -th resource element
$\mathbf{h}^{(\nu,\mu)}$	$[L_{\text{ch}} \times 1]$ : CIR for the receive-transmit antenna pair $(\nu, \mu)$ with path-loss
$\underline{\mathbf{H}}_{n,l}$	$[N_{\text{bs}} \times N_{\text{ue}}]$ : MIMO channel matrix with path-loss at the $(n, l)$ -th resource element
$\underline{\mathbf{P}}$	$[N_{\text{bs}} \times N_{\text{ue}}]$ : path-loss matrix

## Vectors and Matrices in Chapter 3

$\mathbf{H}_{\text{U}_p}^{(\nu,\mu)}$	$[N'_p \times 1]$ : vector of the sampled CTF at the index $u_p^{(\mu)}$ between UE $\mu$ and BS $\nu$
$\hat{\mathbf{H}}_{\text{U}_p}^{(\nu,\mu)}$	$[N'_p \times 1]$ : estimated vector of the sampled CTF at the index $u_p^{(\mu)}$ between UE $\mu$ and BS $\nu$
$\hat{\mathbf{H}}_{\text{U}_p, \text{LIE}}^{(\nu,\mu)}$	$[2 \times 1]$ : estimated CTF vector at the two neighbouring pilot subcarriers
$\mathbf{R}_{\text{U}_p}^{(\nu,\mu)}$	$[N'_p \times 1]$ : pilot observation vector of UE $\mu$ at BS $\nu$
$\mathbf{S}_{\text{U}_p}^{(\nu,\mu)}$	$[N'_p \times 1]$ : non-zero pilot sequence vector of UE $\mu$
$\underline{\mathbf{S}}_{\text{U}_p}^{(\mu)}$	$[N'_p \times N'_p]$ : non-zero pilot sequence matrix of UE $\mu$
$\mathbf{W}_{u_d}^{(\nu,\mu)}$	$[N'_p \times 1]$ : general linear filter for channel estimation of $H_{u_d}^{(\nu,\mu)}$
$\mathbf{W}_{u_d, \text{LIE}}^{(\nu,\mu)}$	$[2 \times 1]$ : linear interpolation and extrapolation filter
$\mathbf{W}_{u_d, \text{WF}, \text{mis}}^{(\nu,\mu)}$	$[N'_p \times 1]$ : mismatched one-dimensional Wiener filtering for channel estimation of $H_{u_d}^{(\nu,\mu)}$
$\mathbf{W}_{u_d, \text{WF}, \text{opt}}^{(\nu,\mu)}$	$[N'_p \times 1]$ : one-dimensional optimal Wiener filtering for channel estimation of $H_{u_d}^{(\nu,\mu)}$
$\mathbf{Z}_{\text{U}_p}^{(\nu,\mu)}$	$[N'_p \times 1]$ : noise vector at the pilot count index $u_p^{(\mu)}$ between UE $\mu$ and BS $\nu$
$\tilde{\mathbf{Z}}_{\text{U}_p}^{(\nu,\mu)}$	$[N'_p \times 1]$ : normalized noise vector at the pilot count index $u_p^{(\mu)}$ between UE $\mu$ and BS $\nu$
$\Phi_{\mathbf{H}_{\text{U}_p}^{(\nu,\mu)} H_{u_d}^{(\nu,\mu)}}$	$[N'_p \times 1]$ : cross-correlation vector between $\mathbf{H}_{\text{U}_p}^{(\nu,\mu)}$ and $H_{u_d}^{(\nu,\mu)}$
$\Phi_{\mathbf{H}_{\text{U}_p}^{(\nu,\mu)} H_{u_d}, \text{mis}}$	$[N'_p \times 1]$ : mismatched form of the cross-correlation vector between $\mathbf{H}_{\text{U}_p}^{(\nu,\mu)}$ and $H_{u_d}^{(\nu,\mu)}$
$\underline{\Phi}_{\mathbf{H}_{\text{U}_p}^{(\nu,\mu)} \mathbf{H}_{\text{U}_p}^{(\nu,\mu)}}$	$[N'_p \times N'_p]$ : auto-correlation matrix of $\mathbf{H}_{\text{U}_p}^{(\nu,\mu)}$

$\underline{\Phi}_{\mathbf{H}_{\mathbb{U}_p} \mathbf{H}_{\mathbb{U}_p}, \text{mis}}^{(\nu, \mu)}$   $[N'_p \times N'_p]$ : mismatched form of the auto-correlation matrix of  $\mathbf{H}_{\mathbb{U}_p}^{(\nu, \mu)}$

## Vectors and Matrices in Chapter 4

$\underline{\mathbf{A}}_{\mu\mu}$   $[L_{\text{ch}} \times L_{\text{ch}}]$ : the  $\mu$ -th block diagonal matrix of  $\underline{\mathbf{F}}_{\text{stack}, p}^H \mathbf{F}_{\text{stack}, p}$

$\underline{\mathbf{B}}_{\mu\mu}$   $[L_{\text{ch}} \times L_{\text{ch}}]$ :  $\underline{\mathbf{B}}_{\mu\mu} = \underline{\mathbf{A}}_{\mu\mu} + \sigma_Z^2 \underline{\Phi}_{\mathbf{h}\mathbf{h}}^{(\nu, \mu)-1}$

$\underline{\mathbf{C}}_{\mathbb{U}_p}^{(\mu)}$   $[N_p \times N_p]$ : cross-correlation matrix between the pilot sequence of UE 1 and UE  $\mu$

$\tilde{\mathbf{C}}_{\mathbb{U}_p}^{(\nu)}$   $[N_{\text{fft}} \times N_{\text{fft}}]$ : expanded version of  $\underline{\mathbf{C}}_{\mathbb{U}_p}^{(\mu)}$

$\mathbf{e}^{(\nu, \mu)}$   $[L_{\text{ch}} \times 1]$ : error vector between true  $\mathbf{h}^{(\nu, \mu)}$  and  $\hat{\mathbf{h}}^{(\nu, \mu)}$

$\mathbf{e}_{\text{stack}}^{(\nu)}$   $[N_{\text{ue}} L_{\text{ch}} \times 1]$ : error vector between true  $\mathbf{h}_{\text{stack}}^{(\nu)}$  and estimated  $\hat{\mathbf{h}}_{\text{stack}}^{(\nu)}$

$\mathbf{E}_{\text{stack}}^{(\nu)}$   $[N_{\text{ue}} N_p \times 1]$ : error vector between true  $\mathbf{H}_{\mathbb{U}_p, \text{stack}}^{(\nu)}$  and estimated  $\hat{\mathbf{H}}_{\mathbb{U}_p, \text{stack}}^{(\nu)}$

$\mathbf{E}_{\mathbb{U}_p}^{(\nu, \mu)}$   $[N_p \times 1]$ : error vector in the frequency domain between true  $\mathbf{H}_{\mathbb{U}_p}^{(\nu, \mu)}$  and estimated  $\hat{\mathbf{H}}_{\mathbb{U}_p}^{(\nu, \mu)}$

$\underline{\mathbf{F}}_p$   $[N_p \times L_{\text{ch}}]$ : submatrix of the DFT matrix  $\underline{\mathbf{F}}$

$\underline{\mathbf{F}}_{\text{stack}, p}$   $[N_p \times N_{\text{ue}} L_{\text{ch}}]$ : stacked DFT matrix

$\tilde{\mathbf{h}}^{(\nu, \mu)}$   $[N_{\text{fft}} \times 1]$ : extended CIR vector of  $\mathbf{h}^{(\nu, \mu)}$

$\mathbf{h}_{\text{stack}}^{(\nu)}$   $[N_{\text{ue}} L_{\text{ch}} \times 1]$ : superposition of the CIR of all active UEs

$\hat{\mathbf{h}}_{\text{stack}}^{(\nu)}$   $[N_{\text{ue}} L_{\text{ch}} \times 1]$ : channel estimate of  $\mathbf{h}_{\text{stack}}^{(\nu)}$

$\mathbf{H}_{\mathbb{U}_p}^{(\nu, \mu)}$   $[N_p \times 1]$ : vector of the sampled CTF at the index  $u_p^{(\mu)}$  between UE  $\mu$  and BS  $\nu$

$\hat{\mathbf{H}}_{\mathbb{U}_p}^{(\nu, \mu)}$   $[N_p \times 1]$ : estimate of  $\mathbf{H}_{\mathbb{U}_p}^{(\nu, \mu)}$

$\hat{\mathbf{H}}_{\mathbb{U}_p, \text{stack}}^{(\nu)}$   $[N_{\text{ue}} N_p \times 1]$ : stacked version of  $\hat{\mathbf{H}}_{\mathbb{U}_p}^{(\nu, \mu)}$  over all UEs

$\mathbf{I}_{\text{leak}}$   $[N_{\text{fft}} \times 1]$ : the first column vector of  $\underline{\mathbf{I}}_{\text{leak}}$

$\underline{\mathbf{I}}_{\text{leak}}$   $[N_{\text{fft}} \times N_{\text{fft}}]$ : circulant matrix of leakage effect

$\underline{\mathbf{I}}_{\text{leak}}^{(\mu)}$   $[N_{\text{fft}} \times N_{\text{fft}}]$ : circulant matrix of leakage effect of UE  $\mu$

$\mathbf{R}_{\mathbb{U}_p}^{(\nu)}$   $[N_p \times 1]$ : pilot observation vector at BS  $\nu$

$\hat{\mathbf{R}}_{\mathbb{U}_p}^{(\nu)}$   $[N_p \times 1]$ : normalized pilot observation vector at BS  $\nu$  w.r.t. the energy per transmitted pilot signal

$\mathbf{t}^{(\nu)}$   $[N_{\text{fft}} \times 1]$ : output signal of the received signal transformed by the IDFT from the frequency domain into the time domain

$\mathbf{t}^{(\nu, \mu)}$   $[N_{\text{fft}} \times 1]$ : the received signal vector in the time domain from UE  $\mu$

$\mathbf{t}_{\text{slc}}^{(\nu, \mu)}$   $[L_{\text{ch}} \times 1]$ : selected signal vector from UE  $\mu$

$\mathbf{T}_{\mathbb{U}_p}^{(\nu)}$   $[N_p \times 1]$ : output of the correlation of the received signal  $\tilde{\mathbf{R}}_{\mathbb{U}_p}^{(\nu)}$  with the pilot sequences of UE 1

$\tilde{\mathbf{T}}_{\text{U}_p}^{(\nu)}$	$[N_{\text{fft}} \times 1]$ : expanded version of $\mathbf{T}_{\text{U}_p}^{(\nu)}$
$\mathbf{V}_{\text{U}_p}^{(\nu)}$	$[N_p \times 1]$ : normalized noise vector at BS $\nu$ w.r.t. the energy per transmitted pilot signal
$\underline{\mathbf{w}}_{\text{slc}}^{(\mu)}$	$[L_{\text{ch}} \times N_{\text{fft}}]$ : selective window for UE $\mu$
$\underline{\mathbf{w}}_{\text{co}}^{(\nu)}$	$[N_{\text{ue}}L_{\text{ch}} \times N_p]$ : linear estimator using CO pilots
$\underline{\mathbf{w}}_{\text{co,sim}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times N_p]$ : simplified channel estimator using CO pilots of $\hat{\mathbf{h}}^{(\nu,\mu)}$
$\underline{\mathbf{w}}_{\text{co,sim,mis}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times N_p]$ : mismatched simplified channel estimator using CO pilots of $\hat{\mathbf{h}}^{(\nu,\mu)}$
$\tilde{\mathbf{Z}}_{\text{U}_p}^{(\nu)}$	$[N_p \times 1]$ : normalized noise vector at BS $\nu$ w.r.t. the energy per transmitted pilot signal
$\underline{\Phi}_{\text{ee}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times L_{\text{ch}}]$ : auto-correlation matrix of $\mathbf{e}^{(\nu,\mu)}$
$\underline{\Phi}_{\text{ee,opt}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times L_{\text{ch}}]$ : optimized value of $\underline{\Phi}_{\text{ee}}^{(\nu,\mu)}$ without model mismatch
$\underline{\Phi}_{\mathbf{e}_{\text{stack}}^{(\nu)} \mathbf{e}_{\text{stack}}^{(\nu)}}$	$[N_{\text{ue}}L_{\text{ch}} \times N_{\text{ue}}L_{\text{ch}}]$ : auto-correlation matrix of $\mathbf{e}_{\text{stack}}^{(\nu)}$
$\underline{\Phi}_{\mathbf{E}_{\text{stack}}^{(\nu,\mu)} \mathbf{E}_{\text{stack}}^{(\nu,\mu)}}$	$[N_{\text{ue}}N_p \times N_{\text{ue}}N_p]$ : auto-correlation matrix of $\mathbf{E}_{\text{stack}}^{(\nu,\mu)}$
$\underline{\Phi}_{\mathbf{E}_{\text{U}_p}^{(\nu,\mu)} \mathbf{E}_{\text{U}_p}^{(\nu,\mu)}}$	$[N_p \times N_p]$ : auto-correlation matrix of $\mathbf{E}_{\text{U}_p}^{(\nu,\mu)}$
$\underline{\Phi}_{\text{hh}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times L_{\text{ch}}]$ : auto-correlation matrix of $\mathbf{h}^{(\nu,\mu)}$
$\underline{\Phi}_{\text{hh,mis}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times L_{\text{ch}}]$ : mismatched form of the auto-correlation matrix of $\mathbf{h}^{(\nu,\mu)}$
$\underline{\Phi}_{\text{hh,opt}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times L_{\text{ch}}]$ : true auto-correlation matrix of $\mathbf{h}^{(\nu,\mu)}$
$\underline{\Phi}_{\text{hT}_{\text{U}_p}}^{(\nu,\mu)}$	$[L_{\text{ch}} \times N_p]$ : cross-correlation matrix of $\mathbf{h}^{(\nu,\mu)}$ and $\mathbf{T}_{\text{U}_p}^{(\nu)}$
$\underline{\Phi}_{\mathbf{h}_{\text{stack}}^{(\nu)} \mathbf{h}_{\text{stack}}^{(\nu)}}$	$[N_{\text{ue}}L_{\text{ch}} \times N_{\text{ue}}L_{\text{ch}}]$ : auto-correlation matrix of $\mathbf{h}_{\text{stack}}^{(\nu)}$
$\underline{\Phi}_{\mathbf{h}_{\text{stack}}^{(\nu,\mu)} \mathbf{T}_{\text{U}_p}^{(\nu)}}$	$[N_{\text{ue}}L_{\text{ch}} \times N_p]$ : cross-correlation matrix of $\mathbf{h}_{\text{stack}}^{(\nu,\mu)}$ and $\mathbf{T}_{\text{U}_p}^{(\nu)}$
$\underline{\Phi}_{\mathbf{H}_{\text{U}_p}^{(\nu,\mu)} H_{\text{U}_p}^{(\nu,\mu)}}$	$[N_p \times 1]$ : cross-correlation vector between $\mathbf{H}_{\text{U}_p}^{(\nu,\mu)}$ and $H_{\text{U}_p}^{(\nu,\mu)}$
$\underline{\Phi}_{\mathbf{H}_{\text{U}_p}^{(\nu,\mu)} \mathbf{H}_{\text{U}_p}^{(\nu,\mu)}}$	$[N_p \times N_p]$ : auto-correlation matrix of $\mathbf{H}_{\text{U}_p}^{(\nu,\mu)}$
$\underline{\Phi}_{\mathbf{T}_{\text{U}_p}^{(\nu)} \mathbf{T}_{\text{U}_p}^{(\nu)}}$	$[N_p \times N_p]$ : auto-correlation matrix of $\mathbf{T}_{\text{U}_p}^{(\nu)}$
$\underline{\Phi}_{\mathbf{V}_{\text{U}_p}^{(\nu)} \mathbf{V}_{\text{U}_p}^{(\nu)}}$	$[N_p \times N_p]$ : auto-correlation matrix of $\mathbf{V}_{\text{U}_p}^{(\nu)}$

## Vectors and Matrices in Chapter 6

$\mathbf{E}_{\text{JD},u_d}$	$[N_{\text{ue}} \times 1]$ : subcarrier specific joint detection error vector
$\underline{\mathbf{E}}_{u_d}$	$[N_{\text{bs}} \times N_{\text{ue}}]$ : channel estimation error matrix
$\underline{\mathbf{H}}_{u_d}$	$[N_{\text{bs}} \times N_{\text{ue}}]$ : MIMO channel matrix for data transmission
$\underline{\hat{\mathbf{H}}}_{u_d}$	$[N_{\text{bs}} \times N_{\text{ue}}]$ : estimate of $\underline{\mathbf{H}}_{u_d}$
$\underline{\mathbf{W}}_{\text{MMSE,CE},u_d}$	$[N_{\text{ue}} \times N_{\text{bs}}]$ : MMSE filter with channel estimation error
$\underline{\mathbf{W}}_{\text{MMSE,PCE},u_d}$	$[N_{\text{ue}} \times N_{\text{bs}}]$ : MMSE filter under perfect channel estimation

$\underline{\mathbf{W}}_{\text{UB,CE},u_d}$	$[N_{\text{ue}} \times N_{\text{bs}}]$ : unbiased MMSE filter with channel estimation error
$\underline{\mathbf{W}}_{\text{UB,PCE},u_d}$	$[N_{\text{ue}} \times N_{\text{bs}}]$ : unbiased MMSE filter under perfect channel estimation
$\underline{\mathbf{W}}_{\text{ZF,CE},u_d}$	$[N_{\text{ue}} \times N_{\text{bs}}]$ : ZF filter with channel estimation error
$\underline{\mathbf{W}}_{\text{ZF,PCE},u_d}$	$[N_{\text{ue}} \times N_{\text{bs}}]$ : ZF filter under perfect channel estimation
$\mathbf{X}_{u_d}$	$[N_{\text{ue}} \times 1]$ : transmitted data signal vector
$\hat{\mathbf{X}}_{u_d}$	$[N_{\text{ue}} \times 1]$ : decision vector of data signal
$\tilde{\mathbf{Y}}_{u_d}$	$[N_{\text{bs}} \times 1]$ : received data signal vector
$\tilde{\mathbf{Z}}_{u_d}$	$[N_{\text{bs}} \times 1]$ : effective noise vector
$\underline{\Phi}_{\mathbf{E}_{\text{JD},u_d} \mathbf{E}_{\text{JD},u_d}}$	$[N_{\text{ue}} \times N_{\text{ue}}]$ : covariance matrix of $\mathbf{E}_{\text{JD},u_d}$
$\underline{\Phi}_{\mathbf{E}_{\text{JD},u_d} \mathbf{E}_{\text{JD},u_d}, \text{UB,CE}}$	$[N_{\text{ue}} \times N_{\text{ue}}]$ : covariance matrix of $\mathbf{E}_{\text{JD},u_d}$ for unbiased MMSE filter with channel estimation error
$\underline{\Phi}_{\mathbf{E}_{\text{JD},u_d} \mathbf{E}_{\text{JD},u_d}, \text{UB,PCE}}$	$[N_{\text{ue}} \times N_{\text{ue}}]$ : covariance matrix of $\mathbf{E}_{\text{JD},u_d}$ for unbiased MMSE filter under perfect channel estimation
$\underline{\Phi}_{\mathbf{E}_{\text{JD},u_d} \mathbf{E}_{\text{JD},u_d}, \text{ZF,CE}}$	$[N_{\text{ue}} \times N_{\text{ue}}]$ : covariance matrix of $\mathbf{E}_{\text{JD},u_d}$ for ZF filter with channel estimation error
$\underline{\Phi}_{\mathbf{E}_{\text{JD},u_d} \mathbf{E}_{\text{JD},u_d}, \text{ZF,PCE}}$	$[N_{\text{ue}} \times N_{\text{ue}}]$ : covariance matrix of $\mathbf{E}_{\text{JD},u_d}$ for ZF filter under perfect channel estimation
$\underline{\Phi}_{\mathbf{X}_{u_d} \mathbf{X}_{u_d}}$	$[N_{\text{ue}} \times N_{\text{ue}}]$ : covariance matrix of $\mathbf{X}_{u_d}$
$\underline{\Phi}_{\tilde{\mathbf{Z}}_{u_d} \tilde{\mathbf{Z}}_{u_d}}$	$[N_{\text{bs}} \times N_{\text{bs}}]$ : covariance matrix of $\tilde{\mathbf{Z}}_{u_d}$



# Introduction

## 1.1 Motivation

Wireless and mobile communications become more popular in the recent years. With the introduction of new applications in the consumer market, such as mobile internet services in smart phones, mobile video and gaming, the number of mobile phone users worldwide has increased a lot, together with the data traffic and the global annual mobile revenue.

This development will involve data-oriented broadband services with the requirement for the higher data rates. Under the consideration of the facts that the mobile bandwidth is a rare resource so that very expensive because of the license auction, it is necessary to improve spectral efficiency over those provided by the current mobile communications systems, like say high speed packet access (HSPA).

Under this background, new technologies are needed to meet this high-speed and fair data-transmission demand correspondingly. And at the same time, it should be achieved at a reasonable cost per bit so that the services provided are still profitable for the mobile operators.

Recently, the standard long term evolution (LTE) specification (Release 8) [3GP09] is finalized by utilizing multiple antennas at base stations and terminal side, termed so-called "multiple-input multiple-output (MIMO)" in the literature. But, since the number of deployable antennas is limited, e.g. the limited size at the terminal side and regulation problems at the base station side, the increase of spectral efficiency and user fairness expected in the LTE specification are also limited.

The latest development in this area is the extension of the LTE specification, coming up with the LTE-Advanced standard (LTE-A), whose main idea focuses on the technologies termed literally as "coordinated multi-point (CoMP)", both at the uplink and downlink direction [Irm09].

With the introduction of the idea of cooperating base stations, we have to deal with the problem of the presence of inter-cell interference. In mobile cellular systems, a frequency-reuse factor of 1 is proposed to improve the spectral efficiency, resulting in the possible

dominate interference from co-channel users outside the main cell. From the information theoretic point of view, this interference causes decreasing the system performance [Kha08, Mar10].

CoMP technologies propose to coordinate the base stations through mutual exchange of the information between them to mitigate or even take advantage of the inter-cell interference. In this thesis, we only consider the uplink scenarios. According to the specifications in LTE-Advanced, there are various main issues to be considered and solved, taking a few important ones as example:

1. the backhaul optimization problem [Kha08];
2. the synchronization problem [KF10a];
3. the channel estimation problem [RF10];
4. and the efforts to close the gap between the theoretical and experimental world [KF10c, Irm09].

## 1.2 Related Work

In the recent years there have a lot of work done on the topics of CoMP uplink. Within the Vodafone chair at Technische Universität Dresden, works are already done by Khattak [Kha08, KF08]. Based on these theoretical investigations, a large-scale testbed is established in the down town of Dresden within the EASY-C project, funded by the Germany government, also the project reports show some valuable findings and results, especially from the implementation and experiment point of view [FR10, KF10c, Irm09]. From Technische Universität Berlin there have also been publications on field trial results for the CoMP uplink in the Berlin testbed [JvH09, JJ10].

Also, research groups in the world investigate some special aspects of the CoMP uplink. Andrews from Texas University (Austin, USA) explores the potential of interference cancellation through the CoMP uplink in [And05] and Shamai from Israel studies the backhaul efficiency of the CoMP uplink from the information theoretic point of view [SS08].

Note that all the above investigations are based on the assumption of perfect channel knowledge for simplicity. To the best of my knowledge, there are only few research groups who have explored the issues related to channel estimation under the context of CoMP uplink [Mar10, Man05]. However, on the one hand, the authors in [Mar10] only model non-perfect channel estimation into the CoMP uplink system for the information-theoretic analysis and system level simulation. On the other hand, the pilot and channel estimator design proposed by Maniatis in [Mar10, Man05] are not compatible with the system parameters of the CoMP uplink specified in the LTE-advanced specification.



## 1.3 Objective of this Thesis

In this thesis, we concentrate on the problem of channel estimation of the CoMP uplink with the system parameters specified in the LTE-advanced specification.

First of all, we try to answer the question what could be the possible design of the specific pilot and channel estimator for the CoMP uplink. The first design is the traditional channel estimator using frequency-orthogonal pilots, while the second one is our new proposed channel estimator using code-orthogonal pilots.

Secondly, we compare the performance of these two channel estimation designs with the analytical analysis and simulations to find out which one is the better option to be implemented in the CoMP uplink.

Finally, this thesis aims to investigate improved filters for joint data detection in the CoMP uplink system under channel estimation error.

## 1.4 Outline of this Thesis

Chapter 2 gives an overview of basic concepts for the analysis of the CoMP uplink system, including the transmitter, channel and receiver model.

Chapter 3 introduces the channel estimation design using frequency-orthogonal pilots. With a running example, the specific pilot and channel estimator design are explained. Furthermore, the performance of channel estimation error is evaluated by the metric of the mean square error (MSE) with the numerical simulations.

Chapter 4 introduces the channel estimation design using code-orthogonal pilots. First, utilizing the properties of the leakage effect, we propose the methods to mitigate the multi-user interference for channel estimation using code-orthogonal pilots. Then, the specific pilot design using the Frank-Zadoff-Chu (FZC) sequences is proposed, together with the corresponding channel estimator. Finally, the performance of channel estimation error is evaluated by the metric of the mean square error (MSE) with the numerical simulations.

Chapter 5 compares the channel estimation design with frequency-orthogonal and code-orthogonal pilots. First, the equivalence of both channel estimation designs is derived. Then, the channel estimation error is appropriately modeled into the CoMP uplink system. Numerical simulations w.r.t. the metric of the effective signal to noise ratio (SNR) degradation before detection are run to verify that the channel estimator using code-orthogonal pilots is the better option. Finally, the specific parameter design of code-orthogonal pilots are discussed.

In Chapter 6 the performance of joint data detection in the CoMP uplink system is evaluated. As the reference, we first introduce the joint data detection filters with perfect channel knowledge. Then, we propose the new detection filters under the consideration of channel estimation error. Finally, the numerical simulations w.r.t. bit error rate (BER) are run to verify the joint data detection performance in the CoMP uplink system.

Finally, Chapter 7 summarizes and discusses the contributions of this thesis. Furthermore, the open topics are mentioned for further research.

## 1.5 Notation

1. In this thesis,  $H$  and  $h$  will be used for channel transfer function (CTF) and channel impulse response (CIR) signals, respectively;
2. Lowercase letters written in italics denote scalar variables. Uppercase letters written in italics denote constant parameters, when there are only texts in the subscript, giving the specific description of the constant parameters, e.g.,  $N_p$ , the number of pilots. For other cases, when there is nothing in the subscript, e.g.,  $H$  or letters in the subscript, e.g.,  $H_i$  and  $H_{\text{stack},i}$ , they denote scalar variables;
3. Abbreviations are also used to denote variables, e.g.,  $SNR$  and  $MSE$ , for values of signal to noise ratio and mean square error, respectively;
4. Boldface letters represent vectors (e.g.,  $\mathbf{H}$  and  $\mathbf{h}$ ), while both boldface and underlined letters represent matrices (e.g.,  $\underline{\mathbf{H}}$  and  $\underline{\mathbf{h}}$ );
5. Italic letters in the subscript and superscript represent variables, normally denoting the entry index, e.g.,  $H_m$ ,  $\mathbf{H}_m$  and  $\mathbf{h}^{(\nu,\mu)}$ . While texts in the subscript give the specific description, e.g.,  $N_{\text{fft}}$  the fast Fourier transform (FFT) size and  $\underline{\mathbf{w}}_{\text{co,opt}}^{(\nu)}$  the optimized linear estimator with the code orthogonal (CO) pilot design;
6. Selecting the  $m$ -th element from a vector is denoted by  $\mathbf{H}[m] = H_m$ , with

$$\mathbf{H} = \begin{bmatrix} H_1 \\ \vdots \\ H_{N_{\text{col}}} \end{bmatrix} \in \mathbb{C}^{N_{\text{col}} \times 1};$$

7.  $\mathbf{H} : [N_{\text{col}} \times 1]$  represents an  $N_{\text{col}} \times 1$  column vector. If not specifically mentioned, all vectors described in this thesis are column vectors.  $\underline{\mathbf{H}} : [N_{\text{col}} \times N_{\text{row}}]$  represents an  $N_{\text{col}} \times N_{\text{row}}$  matrix;
8. Selecting an element from the  $m$ -th row and  $n$ -th column of a matrix is denoted by  $\underline{\mathbf{H}}[m, n] = H_{m,n}$ , with

$$\underline{\mathbf{H}} = \begin{pmatrix} H_{1,1} & \cdots & H_{1,N_{\text{row}}} \\ \vdots & \ddots & \vdots \\ H_{N_{\text{col}},1} & \cdots & H_{N_{\text{col}},N_{\text{row}}} \end{pmatrix} \in \mathbb{C}^{N_{\text{col}} \times N_{\text{row}}}.$$

If there are too many letters and texts in the subscript, some of them can be moved to the superscript, e.g.,  $\underline{\mathbf{H}}_{n,l}[\nu, \mu] = H_{n,l}^{(\nu,\mu)}$ ;

9. Selecting the  $m$ -th row vector from a matrix is denoted by  $\underline{\mathbf{H}}[m, :] = \mathbf{H}_{\text{row},m}$ , while selecting the  $n$ -th column vector is denoted by  $\underline{\mathbf{H}}[:, n] = \mathbf{H}_{\text{col},n} = \mathbf{H}_n$ ;
10. Selecting a sub-matrix, which consists of the  $m_1$ -th to  $m_2$ -th row and the  $n_1$ -th to  $n_2$ -th column from a matrix is denoted by  $\underline{\mathbf{H}}[m_1 : m_2, n_1 : n_2]$ ;
11. Calligraphic bold letters (e.g.,  $\mathbb{U}$ ) refer to sets,  $\emptyset$  refers to the empty set, and e.g.,  $|\mathbb{U}|$  denotes the size or cardinality of a set;
12. The notation  $\mathbf{x} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{m}, \underline{\Phi}_{\mathbf{xx}})$  states that  $\mathbf{x}$  is a vector of complex Gaussian random variables with mean  $\mathcal{E}(\mathbf{x}) = \mathbf{m}$  and covariance matrix  $\mathcal{E}(\mathbf{xx}^H) = \underline{\Phi}_{\mathbf{xx}}$ .

